

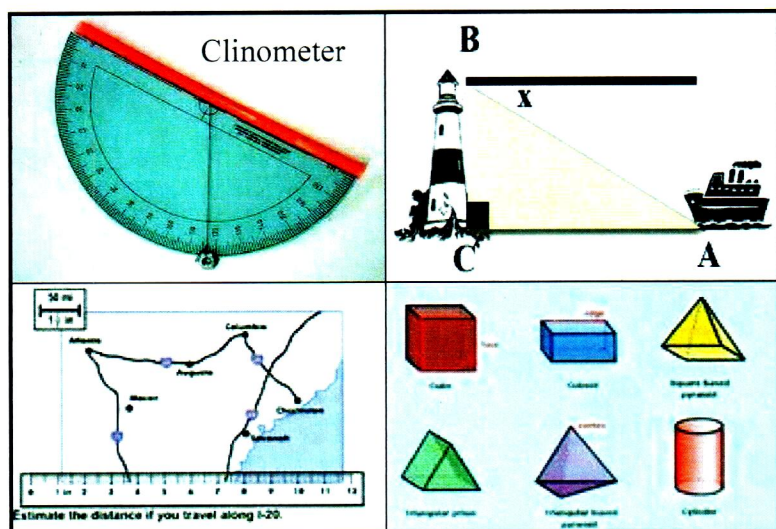


DEPARTMENT OF EDUCATION

GRADE 10

MATHEMATICS

UNIT 6



MEASUREMENT

Published by:



FLEXIBLE OPEN AND DISTANCE EDUCATION
PRIVATE MAIL BAG, P.O. WAIGANI, NCD
FOR THE DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA
2016

MATHEMATICS

GRADE 10

UNIT 6

MEASUREMENT

TOPIC 1: INDIRECT MEASUREMENT

TOPIC 2: SCALE AND SIMILARITY

TOPIC 3: AREA

TOPIC 4: VOLUME

Acknowledgements

We acknowledge the contributions of all Secondary teachers and people who in one way or another helped to develop this Course.

Special thanks is given to the Staff of Mathematics Department of FODE who played active role in coordinating writing workshops, outsourcing lesson writing and editing processes, involving selected teachers from NCD.

We also acknowledge the professional guidance provided by the Curriculum Development and Assessment Division throughout the processes of writing and, the services given by the members of the Mathematics Subject Review and Academic Committees.

MR. DEMAS TONGOGO
Principal-FODE



Flexible Open and Distance Education
Papua New Guinea

Published in 2016

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ISBN: 978-9980-89-715-2

National Library Services of Papua New Guinea

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a/CO- Mathematics Department

Printed by the Flexible, Open and Distance Education

CONTENTS

	Page
Contents.....	3
Secretary's Message.....	4
Strand Introduction.....	5
Study Guide.....	6
TOPIC 1: INDIRECT MEASUREMENT.....	7
☐ Lesson 1: Indirect Measurements.....	8
☐ Lesson 2: Use of a Clinometer.....	19
☐ Lesson 3: Problems on Heights and Distances.....	26
☐ Lesson 4: Bearings	36
☐ Lesson 5: Problems on Bearing	45
☐ Lesson 6: Mixed Problems.....	50
Summary.....	53
Answers to Practice Exercises 1-6.....	54
TOPIC 2: SCALE AND SIMILARITY.....	59
☐ Lesson 7: Scale Models.....	61
☐ Lesson 8: Plans.....	69
☐ Lesson 9: Maaps.....	76
☐ Lesson 10: Enlargements and Similar Shapes.....	86
☐ Lesson 11: Area of Similar Shapes.....	92
Summary.....	96
Answers to Practice Exercises 7-11.....	97
TOPIC 3: AREA.....	99
☐ Lesson 12: Area.....	101
☐ Lesson 13: Surface Area of Prisms: Cubes and Cuboids.....	108
☐ Lesson 14: Surface Area of Triangular and Trapezium Prisms.....	116
☐ Lesson 15: Surface Area of Pyramids.....	122
☐ Lesson 16: Surface Area of Cylinders.....	129
☐ Lesson 17: Surface Area and Problem Solving.....	134
Summary.....	140
Answers to Practice Exercises 12-17.....	141
TOPIC 4: VOLUME.....	145
☐ Lesson 18: Volume of Prisms with Quadrilateral End Faces.....	147
☐ Lesson 19: Volume of Triangular Prisms.....	153
☐ Lesson 20: Volume of Cylinders.....	157
☐ Lesson 21: Volume of Pyramids and Cones.....	162
☐ Lesson 22: Volume and problem Solving.....	167
Summary.....	173
Answers to Practice Exercises 18-22.....	174
REFERENCES.....	175

SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

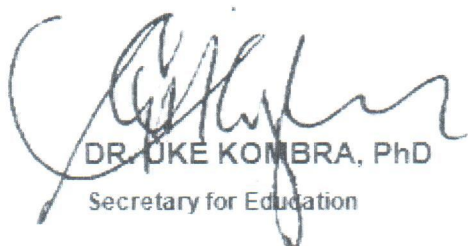
Flexible, Open and Distance Education curriculum is guided by the Department of Education's Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

It is our vision that Papua New Guineans' harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course.



DR. DUKE KOMBRA, PhD
Secretary for Education

UNIT 6: MEASUREMENT



Dear student,

This is the sixth unit of the Grade 10 Mathematics Course. This unit is based on the National Department of Education Lower Secondary Syllabus and Curriculum Framework for Grade 10.

This unit consists of four Topics:

Topic 1: Indirect Measurement

Topic 2: Scale and Similarity

Topic 3: Area

Topic 4: Volume

Topic 1 – Indirect Measurement – You will identify situations where patterns and measurement are applied in both tradition and modern ways. You will also interpret, analyse and solve measurement problems and justify selections and applications of formulae

Topic 2 – Scale and Similarity – You will describe and analyse how scale models are used in parallel to the measurement of the actual object.

Topic 3 – Area – You will revise area of regular and composite shapes. You will also learn to calculate their surface areas and apply these skills in solving problems.

Topic 4 – Volume – You will learn to calculate the volume of prisms, cylinders, cones and pyramids.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise for you to complete. The answers to practice exercises are given at the end of each topic.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy studying this book.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps below and work through the lessons.

- Step 1: Start with Topic 1 and work through it in order.
- Step 2: When you complete Lesson 1, do Practice Exercise 1.
- Step 3: After you have completed each exercise, correct your work. The answers are given at the end of each Topic.
- Step 4: Then revise the lesson and correct your mistakes if any.
- Step 5: When you have completed all these steps, tick the check-box for Lesson 1 on the Content page like shown:

☒ Lesson 1: Points, Lines and Angles

Then go on to the next lesson. Repeat this process, until you complete all the lessons in a Topic. When this is done, revise using the review section.

As you complete each lesson tick the box for that lesson, on the Contents page, like this ☒. This helps you to check on your progress.

Assignment: Topic Tests and Unit Test.

When you have completed all the lessons in a Topic do the Topic Test for that Topic in your Assignment Book. The Unit book tells you when to do this.

When you have completed all Topic Tests for a Unit, revise well and do the Unit Test. The Assignment Book tells you when to do the Unit Test.

Your Distance Teacher will mark the Topic Tests and the Unit Test in each Assignment. The marks you score in each Assignment will count towards your final result. If you score less than 50%, you will repeat the Assignment.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and ensure that you pass all the Assignments.

TOPIC 1

INDIRECT MEASUREMENT

Lesson 1:	Indirect Measurements
Lesson 2:	Use of a Clinometer
Lesson 3:	Problems on Heights and Distances
Lesson 4:	Bearings
Lesson 5:	Problems on Bearings
Lesson 6:	Mixed Problems

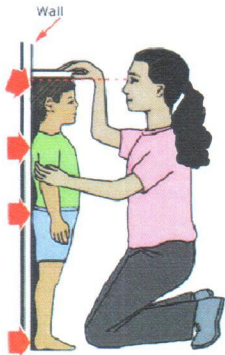
TOPIC 1: INDIRECT MEASUREMENT

Introduction



In your previous studies of Mathematics you have experienced measuring objects, solving for the perimeter, getting the area of plane figures such as polygons and the like. All these involve direct measurements.

In direct measurements, one can easily get the length, height, size and quantitative attributes of an object by placing a ruler, tape measure, protractor and other devices or tools alongside or directly on the object being measured.



Suppose you want to get the height of a child, a tape measure or an extended ruler commonly known as a growth chart in some pediatric clinic is usually placed on the wall and you let the child stand alongside of it to get his accurate height either in centimeters or in inches.

How about getting the height of a tree?
Yes you can do the same technique yet it would not be that easy, right?

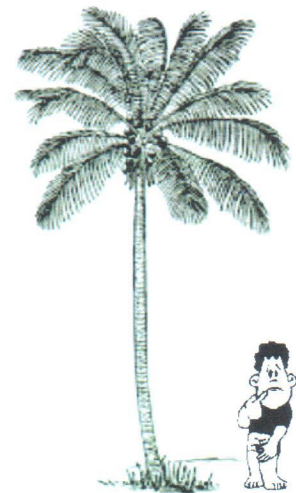
Suppose the tree that you want to measure is an old coconut tree, how many tape measures will you need to accurately measure it? To add, doing so will be quite dangerous, you might fall while you measure it.

With that, is it still possible to get an accurate measure of tall objects like trees and buildings?

This topic will give you a broader knowledge on how to get accurate measures of tall and huge objects in an indirect way.

Is that possible?

Definitely, this topic offers comprehensive discussions on angles, bearings and useful tools which will help you learn to utilize indirect measurement and describe accurately the distance and directions of objects from one place to another.



Lesson 1: Indirect Measurements



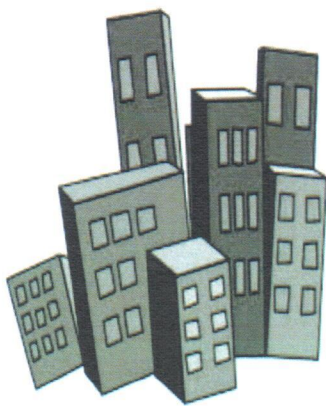
You learnt about basic ways of measurement and the use of measuring tools in your previous Mathematics.



In this lesson, you will:

- explain the meaning of Indirect Measurement
- solve problems on angle of elevation and depression.

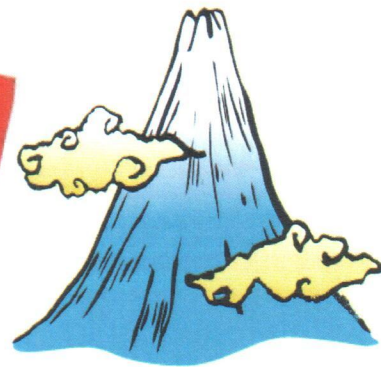
Have you ever thought how the following are measured?



height of buildings



height of the flag poles standing



height of a mountain

Measuring the above mentioned structures looks more challenging than measuring the length, height and width of small objects. Ruler and other measuring tools such as tape measures, meter stick and protractors used in direct measurement of objects would not be as useful in this case.

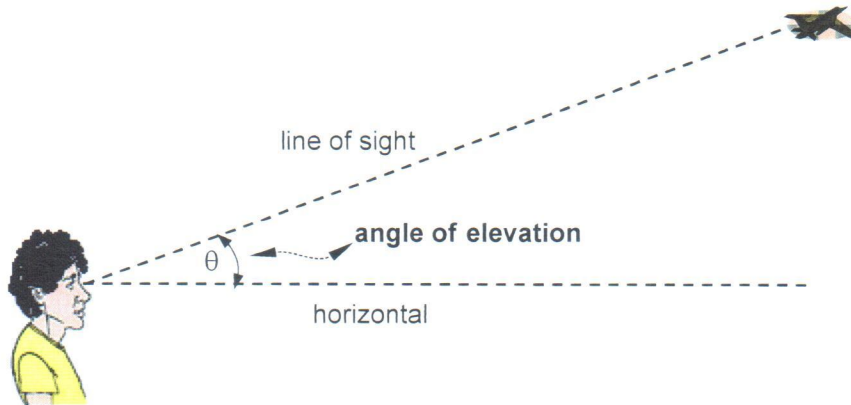
In this lesson you will learn the useful applications of “how to measure things indirectly”. Yes, you can measure things indirectly without touching it.

Indirect Measurement - is a technique that uses proportions to find a measurement when direct measurement is not possible.

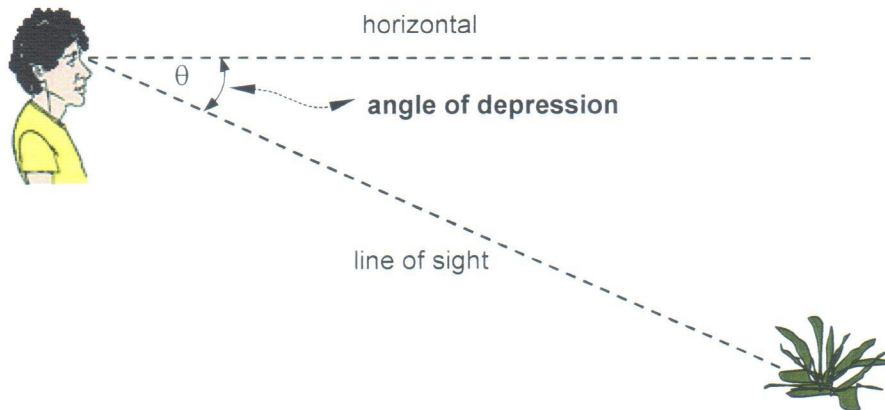
Indirect measurement is possible with our knowledge of trigonometry discussed in Unit 5 combined with your knowledge of the angles of **elevation** and **depression**.

The **angle of elevation** is the angle formed by the horizontal and the line of sight above it.

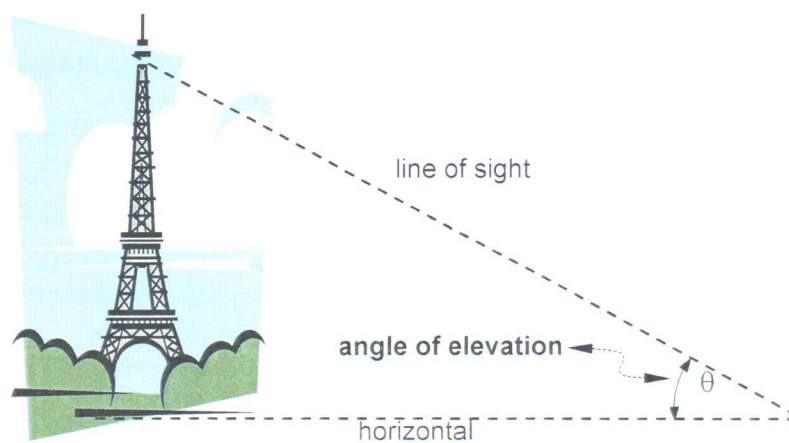
Observe the figure below:



The **angle of depression** is the angle formed by the horizontal and the line of sight below it.



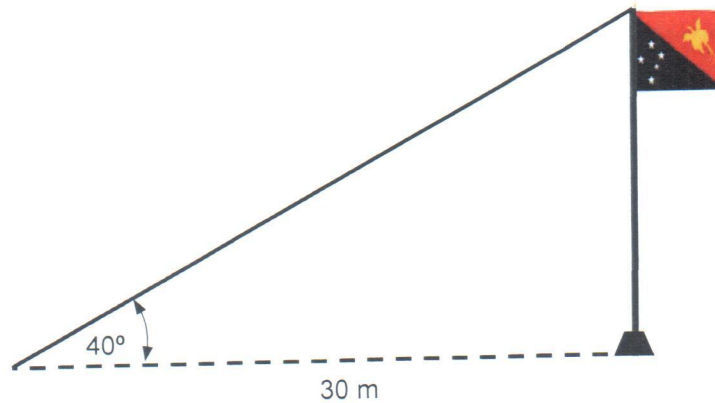
Angles of elevation and depression can also be used in the following instances:



Example 1

The angle of elevation of the top of flag pole at a point 30 meters from its base is 40° . Draw an illustration and find the height of the flag pole.

Solution:



After sketching the figure and labeling it properly, notice that a right triangle is formed.

How did you know that the triangle is right?

Brilliant if you say that the flag pole is erected perpendicular to the ground. In this case we consider the ground as the horizontal; therefore a right angle is formed.

Now notice that the unknown height of the flag pole represented by x and the given 30 meters are the opposite and adjacent sides of the angle of elevation. Does the phrase “**opposite over adjacent**” ring a bell?

Yes! We will use your previous knowledge in trigonometry in Unit 5. (You may refer to the trigonometric table in unit 5 book or simply use a scientific calculator for the proceedings).

Since we know that the opposite and adjacent sides are used. We now use the tangent function and we form:

$$\tan 40^\circ = \frac{x}{30\text{m}}$$

Cross multiply, we get:

$$\begin{aligned} x &= (\tan 40^\circ) \times (30\text{m}) \\ &= (0.8390) \times (30\text{m}) \end{aligned}$$

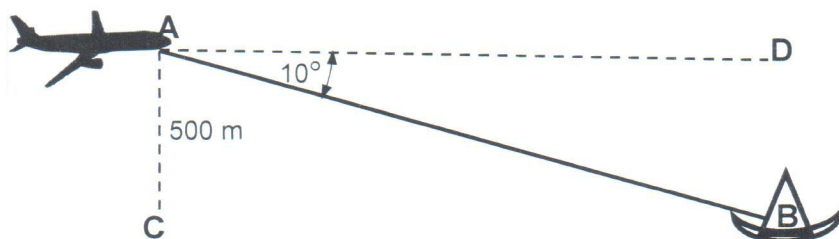
Simplify:

$$x = 25.17 \text{ m}$$

Therefore, the height of the flag pole is 25.17 m.

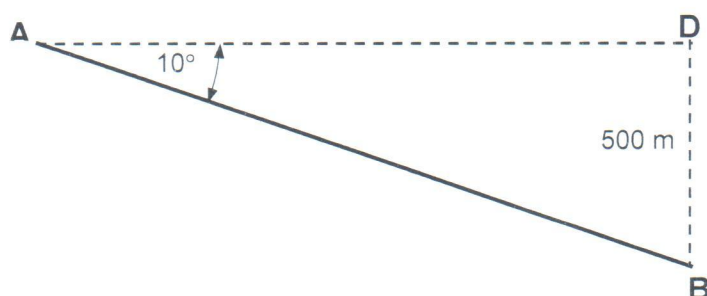
Example 2

A pilot flying a small airplane (A) 500 m above the ground observes a boat (B) at an angle of depression of 10° . What is the horizontal distance of the airplane to the boat? Show an illustration.

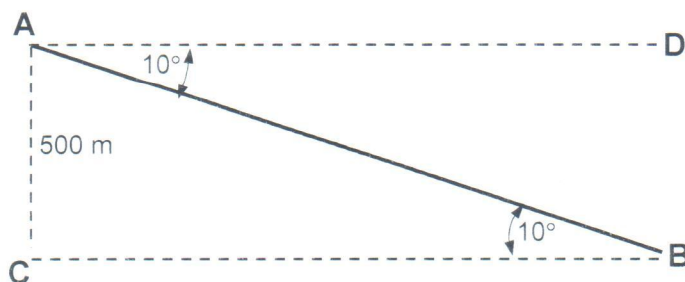


Notice the distance between these pairs of points: $AC = BD$ and $AD = CB$. Having equal distances, we can re-label the figure in two ways to form a right triangle.

First, we can use $\triangle ADB$ where $\angle D$ is a right angle as shown below:



Or it could also be this way, using $\triangle ACB$ where $\angle C$ is a right angle as shown below:



Wondering how the measures of $\angle DAB$ and $\angle ABC$ became equal?

That is a brilliant observation!

Recall in Unit 5, we discussed about the angle relationships of parallel lines cut by a transversal. Since $AD \parallel CB$ and segment AB is considered as a transversal line, we can say that $\angle DAB$ and $\angle ABC$ are congruent or they have equal measures because they are alternate interior angles.

Whichever triangle you wish to use will lead you to the same equation. That is using the Tangent Ratio.

Since we know that the **opposite side** (500 m) and **adjacent** (x) are used.

We now use the tangent function and we form: $\tan 10^\circ = \frac{500\text{m}}{x}$

Cross multiply, we get:

$$x = \frac{500\text{m}}{\tan 10^\circ}$$

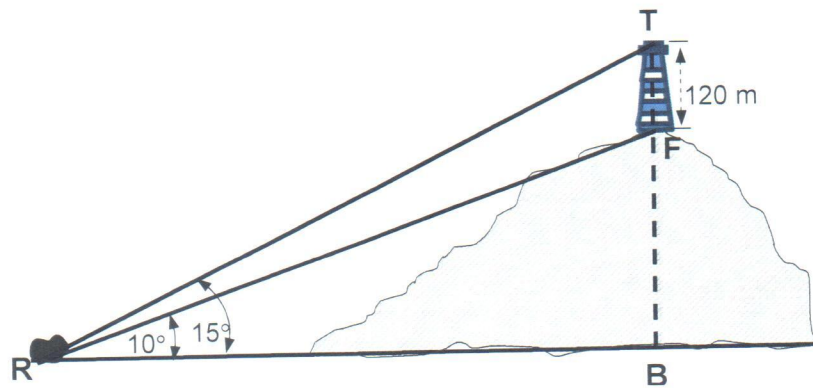
Simplify:

$$x = 2836 \text{ m}$$

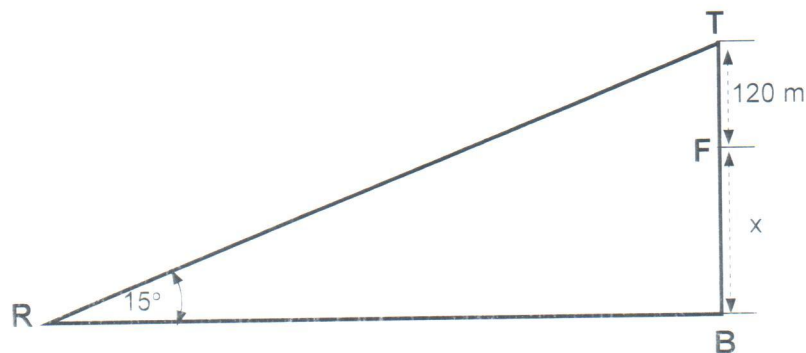
Therefore, we now say that the horizontal distance of the plane to the boat is 2836 m.

Example 3

A lighthouse 120 m high stands at the top of a hill. From a rock (R) down the hill, the angle of elevation of the top of the lighthouse (T) is 15° while the angle of elevation of its foot (F) is 10° . What is the height of the hill?



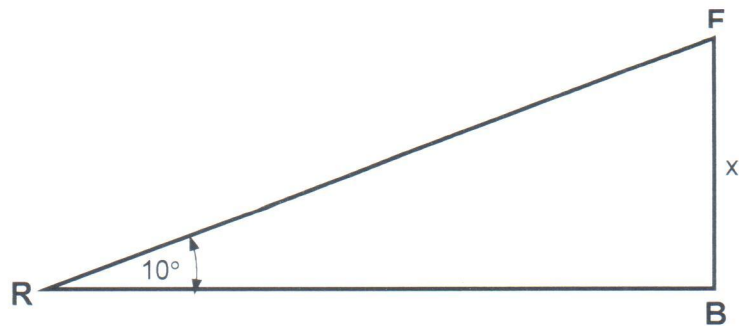
Notice that there are two right triangles formed. $\angle TRB$ with 15° angle and $\angle FRB$ with 10° angle. We will represent the height of the hill FB as x. Using $\angle TRB$ below



We can now form the tangent ratio: $\tan 15^\circ = \frac{120 + x}{RB}$

Cross multiply, we get: $RB = \frac{120 + x}{\tan 15^\circ}$

Using $\angle FRB$ below:



We can now form the tangent ratio: $\tan 10^\circ = \frac{x}{RB}$

Cross multiply, we get: $RB = \frac{x}{\tan 10^\circ}$

Now, we have the two ratios equal to RB

$$RB = \frac{120 + x}{\tan 15^\circ} \quad \text{and} \quad RB = \frac{x}{\tan 10^\circ}$$

Through substitution, we can now equate the two:

$$\frac{120 + x}{\tan 15^\circ} = \frac{x}{\tan 10^\circ}$$

Cross multiply, we get:

$$(\tan 10^\circ) (120 + x) = x (\tan 15^\circ)$$

Simplify using distributive property:

$$[(\tan 10^\circ)(120)] + [(\tan 10^\circ)(x)] = x (\tan 15^\circ)$$

Perform the indicated operations:

$$[(0.1763) (120)] + [(0.1763)(x)] = x (0.2679)$$

$$21.156 + 0.1763x = 0.2679x$$

Transpose all terms with x on the left and constant terms on the right

$$0.1763x - 0.2679x = -21.156$$

$$-0.0916x = -21.156$$

Divide both sides by -0.0916

$$\frac{-0.0916}{-0.0916} x = \frac{-21.150}{-0.0916}$$

$$x = 230.96 \text{ m}$$

Therefore the value of x which is the height of the hill is 230.96 m.

Did you notice that the three illustrative examples we have just solved show some practical applications of trigonometry discussed in Unit 5? Did you also notice that problem number 3 also made use of your knowledge in algebra learnt in your lower grades?

These are just a few of the many practical applications of Mathematics in our lives, and as you go through this unit, you will uncover more of the beauty and practical uses of Mathematics.

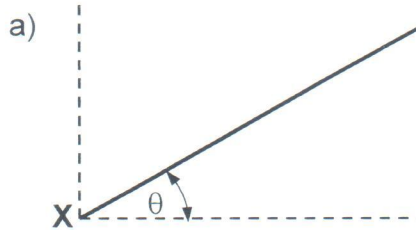
Now, get your pencil and try to do the following exercises.

NOW DO PRACTICE EXERCISE 1



Practice Exercise 1

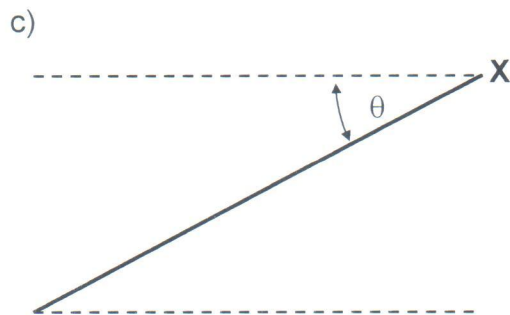
1. Determine if the given angle from point **X** in the figure represented by θ is an angle of elevation or an angle of depression. If the angle is not considered as elevation nor depression, write NA (not applicable). Write the answer on the blank provided below the figure.



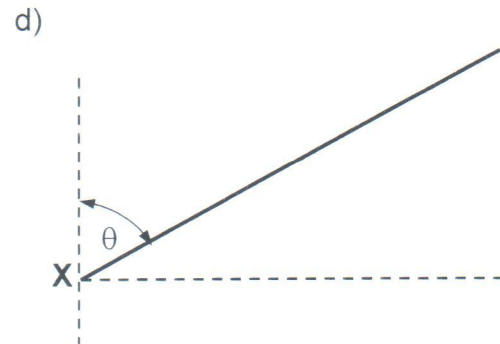
Answer: _____



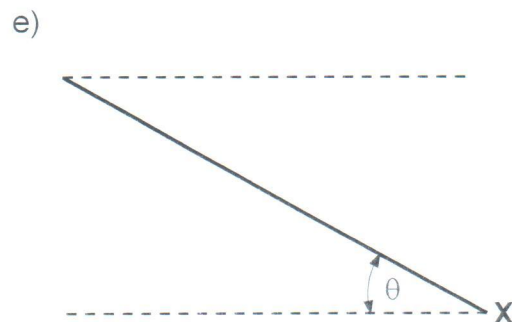
Answer: _____



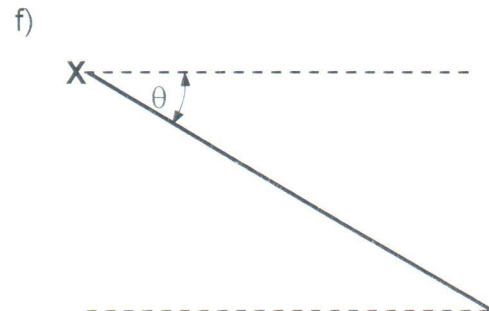
Answer: _____



Answer: _____



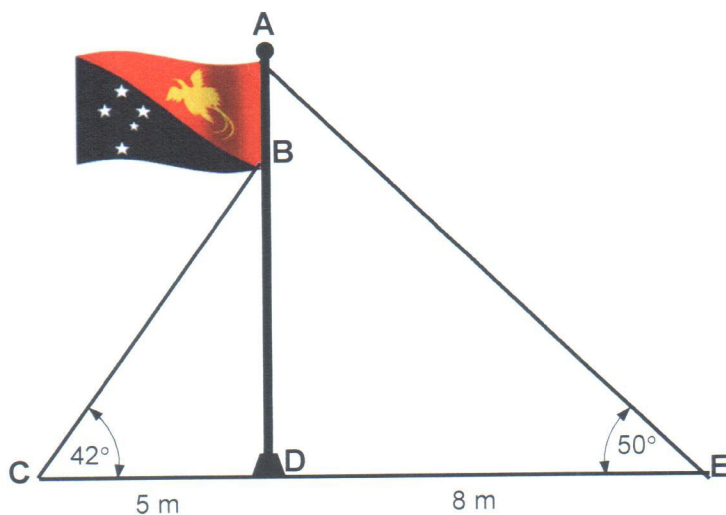
Answer: _____



Answer: _____

2. Use the problem and illustration below to answer the following questions.

The figure below shows a flagpole which is supported by two wires AE and BC tied on the ground.



- a) What is the length of wire BC?

- b) What is the length of wire AE?

- c) How high did wire BD reach?

- d) What is the height of the flagpole?

- e) What is the width of the flag?

3. Solve the following problems. Show your illustration and solution (working out).

- a) A kite is tied up on a stick buried on the ground with an angle of elevation of 30° . If the kite used a thread 45 meters long, how high did the kite reach?

- b) The angle of depression of a boat from a tower is 20° . If the tower is located 70 feet above sea level, what is the horizontal distance of the boat from the tower?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 2: Use of a Clinometer



You learnt about indirect measurements in the previous Lesson.



In this lesson, you will:

- make a clinometer and use it to find angles of elevation and depression
- read the angle of elevation or depression shown in a clinometer.

In Lesson 1, we used the angles of elevation and depression to measure things indirectly. Have you ever wondered how the angles of elevation and depression are obtained?

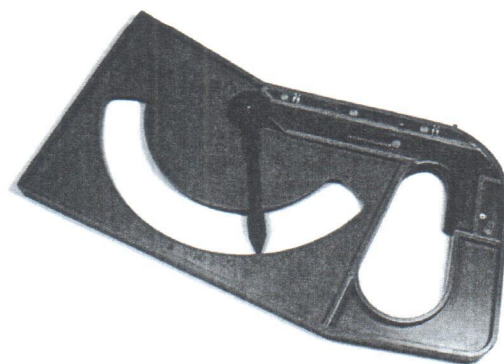
In this Lesson, we will improvise a tool which will help us get the angles of elevation and depression.

Does it sound interesting?

We will do a little cutting and pasting later to come up with this simple yet very helpful tool.

Clinometer

A **clinometer** is a tool or an instrument used to measure angles of elevation and depression. This simple yet very useful tool will help us solve for the height of buildings, trees and other tall objects. We can also use this to compute for the distance of a certain thing with an elevated place.



An improvised clinometer can be made using appropriate materials that could be found around us. Take note that it should have the following essential components: **scale, line of sight and plumb line.**

How to Make a Clinometer?

First look for available materials. Below are the listed materials to be used:

1 piece protractor (scale)

If there is no available protractor, you may draw one on a piece of cardboard as a substitute.

1 piece weight (plumb line)

Any available weight where you can tie on a string

20 cm string (plumb line)

If a bigger protractor is used, make sure that the string is longer than the protractor.

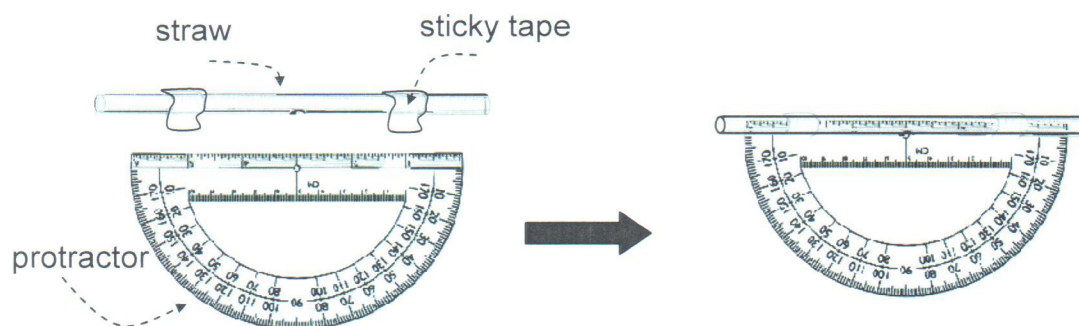
1 piece drinking straw (line of sight)

Sticky tape

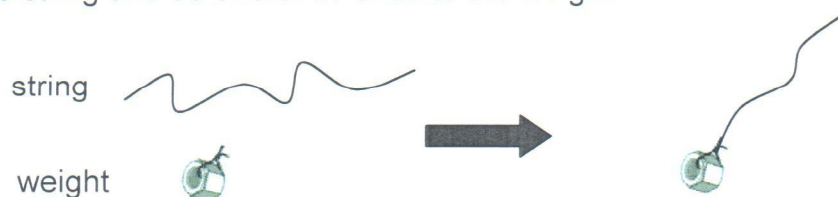
It is recommended to use a transparent sticky tape so details on the protractor will not be covered.

Steps in making a Clinometer:

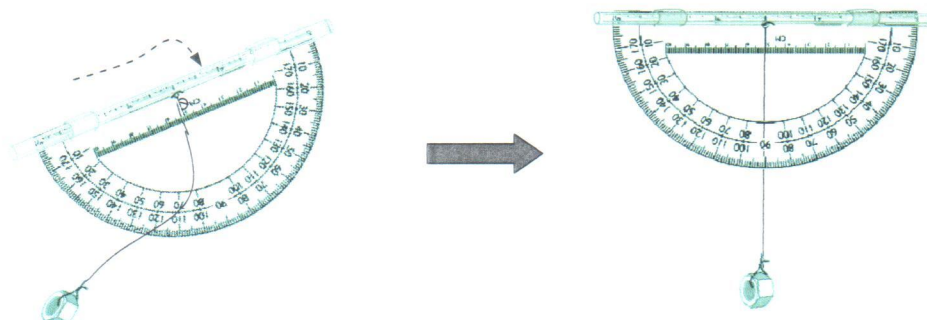
1. Using a sticky tape, attach the straw on the straight edge of the protractor.



2. Get the string and tie one of its ends to the weight.



3. Punch a hole on the center of the straight edge of the protractor and tie the other side of the string on it. Turn the protractor upside down. The weight should pull down the string so that it passes through the degree measures when the clinometer is turned.

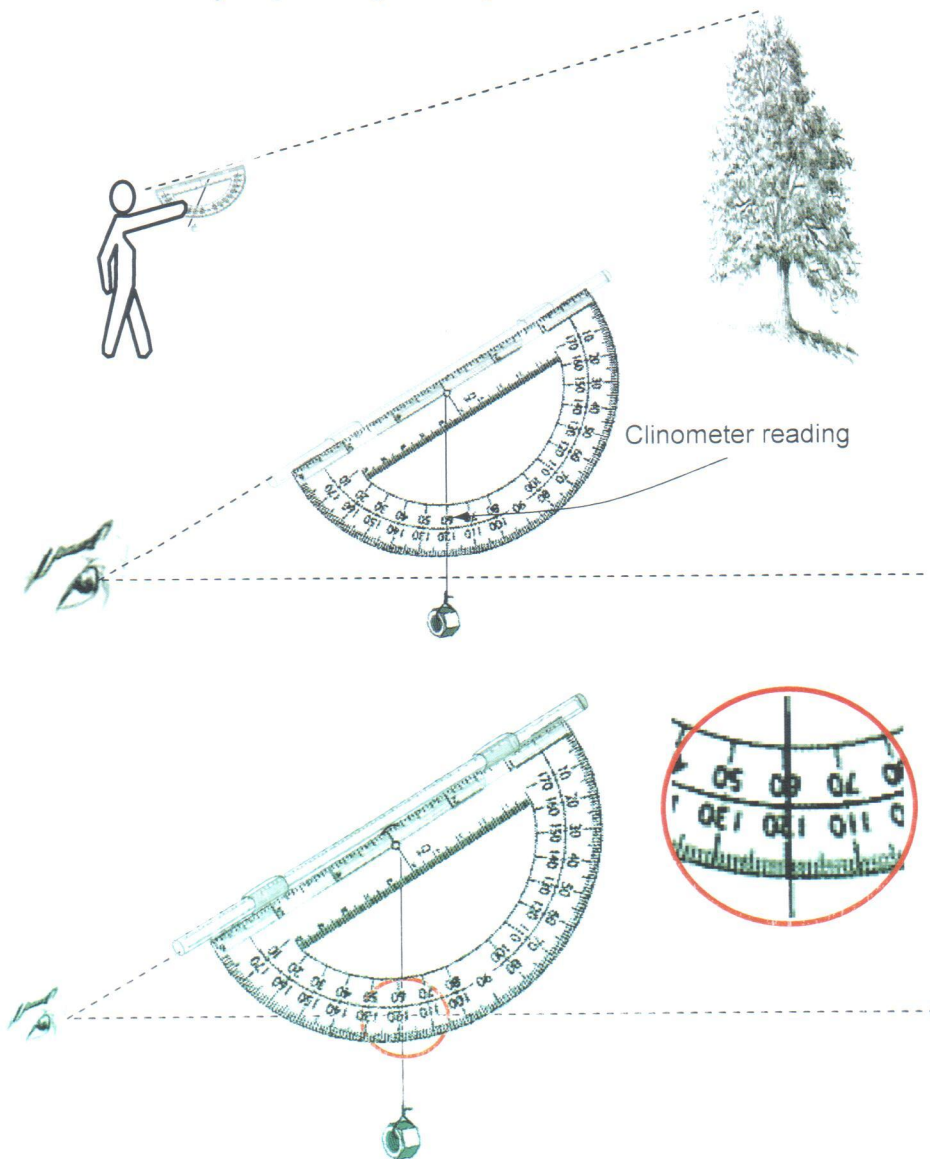


How to Use the Clinometer

1. Stand on a level ground. Hold the Clinometer in such a way that the straw forms a horizontal line and the string points out to 90° .



2. Gently incline the Clinometer and place the straw near the eye. Use the straw to point the tip of the object being measured. You may use the hole to see through it for more accuracy of pointing the object or its tip.



The clinometer points through 60° and 120° . So what is the angle of elevation now?

Notice that when the clinometer is in its base position, meaning the straw is parallel to the horizontal, the reading points at 90° . When the clinometer was tilted or set to an inclined position pointing the tip of the object measured, it points through 60° and 120° . This means that we will only get the measure in between any of the two measures from 90° .

Suppose we want to use the inner scale which reads 60° . So we have $90^\circ - 60^\circ = 30^\circ$. Using the outer scale we have 120° . Getting its difference with 90° , we have $120^\circ - 90^\circ = 30^\circ$. Therefore, we can now say that the angle of elevation derived from the clinometer is 30° .

Example 1

Read the angle of elevation shown in the clinometer.



Solution:

On the Clinometer, the reading shows 30° (inner scale) and 150° (outer scale).

Using the inner scale:

$$90^\circ - 30^\circ = 60^\circ$$

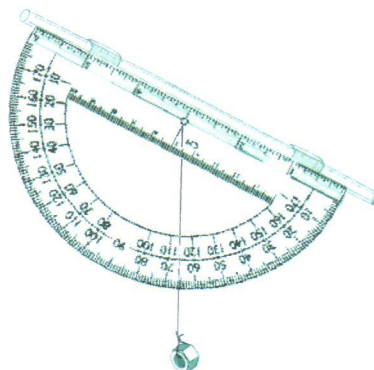
Using the outer scale:

$$150^\circ - 90^\circ = 60^\circ$$

Therefore, the angle of elevation in the Clinometer is 60° .

Example 2

Read the angle of depression shown in the Clinometer.



Solution:

On the Clinometer, the reading shows 115° (inner scale) and 65° (outer scale).

Using the inner scale:

$$115^\circ - 90^\circ = 25^\circ$$

Using the outer scale:

$$90^\circ - 65^\circ = 25^\circ$$

Therefore, the angle of depression in the Clinometer is 25° .

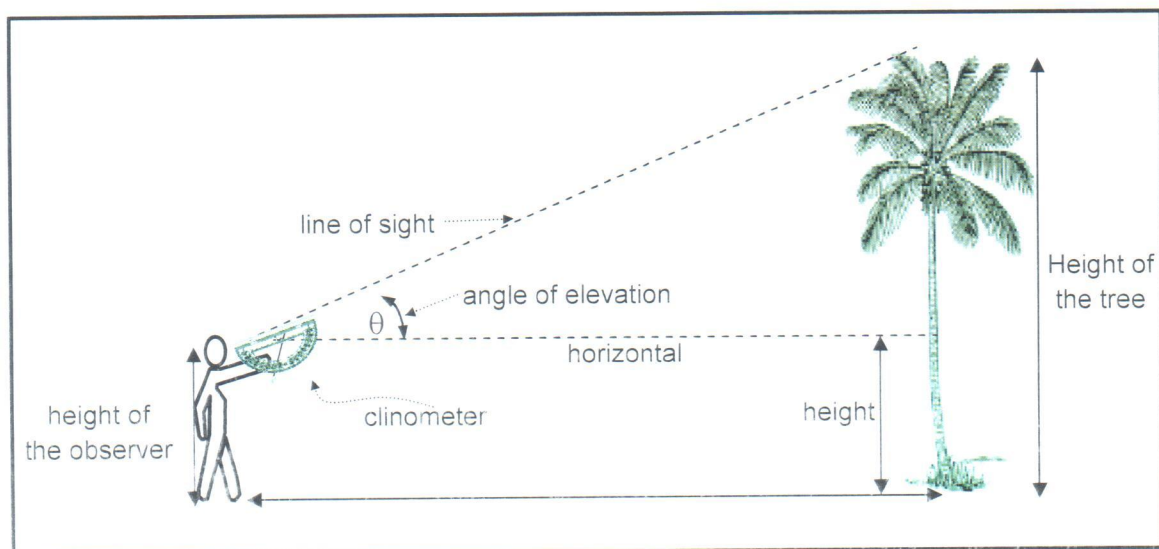
Note:

- If the Clinometer reading (r) is less than 90° , subtract it from 90° or remember the formula $90^\circ - r$.
- If the Clinometer reading (r) is greater than 90° , subtract 90° from it or remember the formula $r - 90^\circ$.

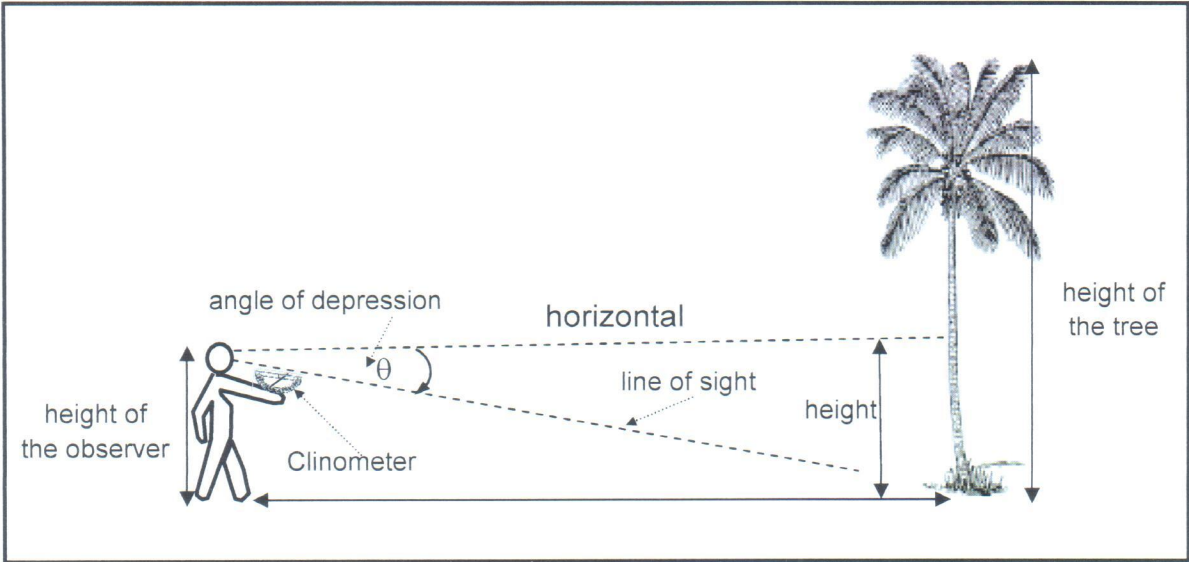
Now you know how to read Clinometer, let us now use it to measure the height of objects.

Below is a sketch of a situation showing the proper way of getting the angle of elevation and depression using a Clinometer.

Measuring Angle of Elevation



Measuring Angle of Depression

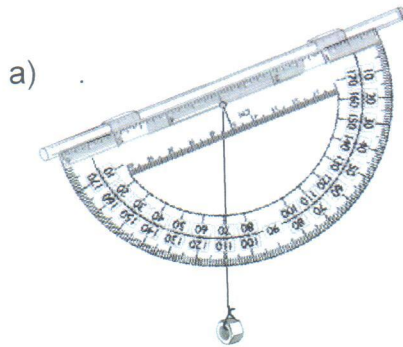


NOW DO PRACTICE EXERCISE 2

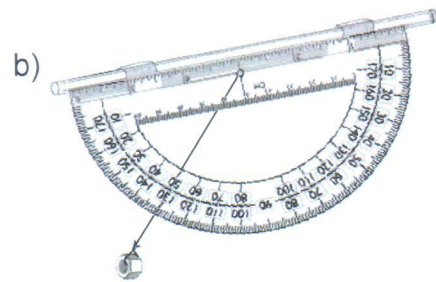


Practice Exercise 2

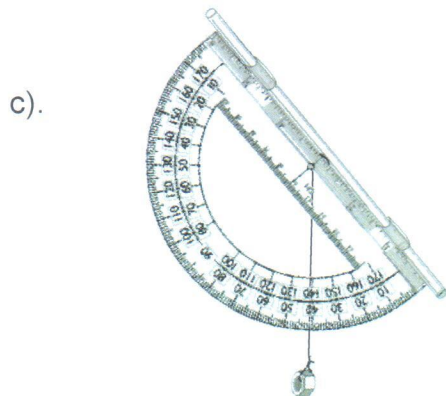
1. Determine if the measure given in a Clinometer is an angle of elevation or depression. Then give the measure of the angle.



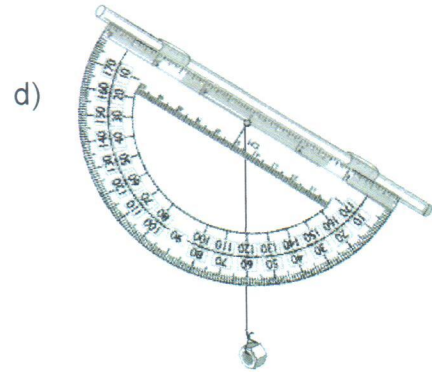
Answer: _____



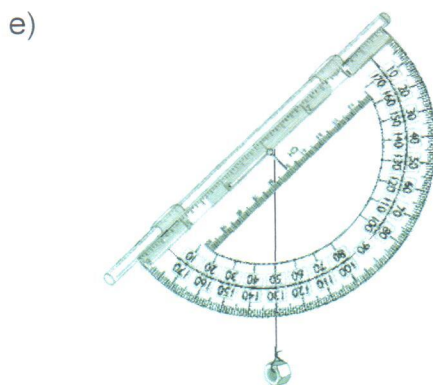
Answer: _____



Answer: _____



Answer: _____



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 3: Problems on Heights and Distances



You learnt about angles of elevation and depression in the previous lesson.



In this lesson, you will:

- use angles of elevation and depression to solve problems on heights and horizontal distances.

Solving for the Heights of Objects

There are several ways presented on how to solve for the heights of objects. One of them is by using a scale ratio which could be very time consuming. With the help of your knowledge in getting the angles of elevation and depression, you can now easily solve for the heights of objects by using the Tangent Ratio.

In unit 5, the Tangent Ratio was discussed. We recall by using the acronym **SOH-CAH-TOA**.

Can you still recall them?

Great if you do!

From the acronym **SOH-CAH-TOA**, **TOA** there represents the Tangent Ratio which mean

$$(T) \text{ tangent } \theta = \frac{(O) \text{ opposite side}}{(A) \text{ adjacent side}}$$

In Lesson 1 of this Unit Book, an introduction of the usefulness of the use of the angles of elevation and depression were discussed. Some examples were also given to show their practical applications.

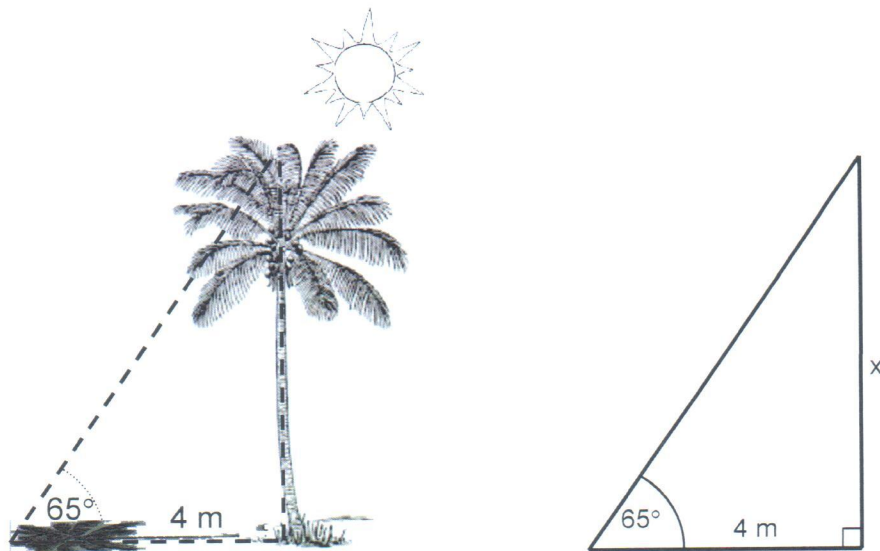
At this point, our focus is on solving for the heights of objects, structures or any elevated objects which could not be measured directly. In addition, the use of the angles of elevation and depression in relation to the Tangent Ratio will be utilized.

Let us try to analyse the following problem.

Example 1

A tree makes a four-meter shadow with the ground when the angle of elevation of the sun from the ground measures 65° . How tall is the tree?

Solution:



The figure on the left is a sketch of the situation given in the problem while the one on the right is a simplified sketch based on the first figure for ease in analysis and computation.

We can say that the tree is standing perpendicular to the ground and therefore we conclude that we have a right triangle.

Analysing the given data in the problem, 4 m. is the side adjacent to the angle of elevation (65°) and the height of the tree represented by x is at the opposite side of the angle.

With that, the Tangent Ratio can be used.

$$\tan \theta = \frac{\text{height of an object}}{\text{horizontal distance to object}}$$

We cross multiply

$$(\tan \theta)(\text{horizontal distance to object}) = \text{height of an object}$$

We simplify by interchanging the left and right sides of the equation

$$\text{height of an object} = (\tan \theta)(\text{horizontal distance to object})$$

Therefore, the simplest way to get the height of an object given the angle of elevation or depression is simplified in the formula:

$$\text{height of an object} = (\tan \theta)(\text{horizontal distance to object})$$

where : θ is the angle of elevation or depression

Now using the derived formula, we now solve the problem on the next page.

We identify the given:

$$\theta = 65^\circ,$$

horizontal distance to the tree / length of the shadow = 4 m

Use the formula and substitute the given in the formula:

$$\text{Height of the tree} = (\tan\theta)(\text{length of the shadow})$$

$$\text{Height of the tree} = (\tan 65^\circ)(4\text{m})$$

Simplify:

$$\text{Height of the tree} = \mathbf{8.58 \text{ m}}$$

Therefore, the height of the tree is 8.58 m.

Example 2

A surveyor measures a distance of 15 m from the base of a building. At that point the angle of elevation to the top of the building is found to be 75° . What is the height of the building?

Solution:

Since we already made a simplified formula in solving for the heights of objects, we can now analyse and solve this problem without making a sketch.

Identify the given:

$$\theta = 75^\circ$$

horizontal distance from the building = 15 m

Use the formula and substitute the given in the formula

$$\text{Height of the building} = (\tan\theta)(\text{horizontal distance from the building})$$

$$\text{Height of the building} = (\tan 75^\circ)(15 \text{ m})$$

Simplify:

$$\text{Height of the building} = \mathbf{55.98 \text{ m}}$$

Therefore, the height of the building is 55.98 m.

It is a lot easier now solving for the height of objects, right?

Just follow a simple formula with a set of simple steps and you will be solving like a pro!

Example 3

A man at the light house gave a signal to the ship approaching the vicinity. The captain of the ship learned that his ship is approximately 18.25 km away from the light house whose angle of depression to the ship is 40° . How high is the man on the lighthouse from the sea level?

Identify the given:

$$\theta = 40^\circ$$

$$\text{horizontal distance from the lighthouse} = 18.25 \text{ km}$$

Use the formula and substitute the given in the formula

$$\begin{array}{l} \text{Height of the lighthouse} \\ \text{from sea level} \end{array} = (\tan \theta)(\text{horizontal distance from the building})$$

$$\begin{array}{l} \text{Height of the lighthouse} \\ \text{from sea level} \end{array} = (\tan 40^\circ)(18.25 \text{ km})$$

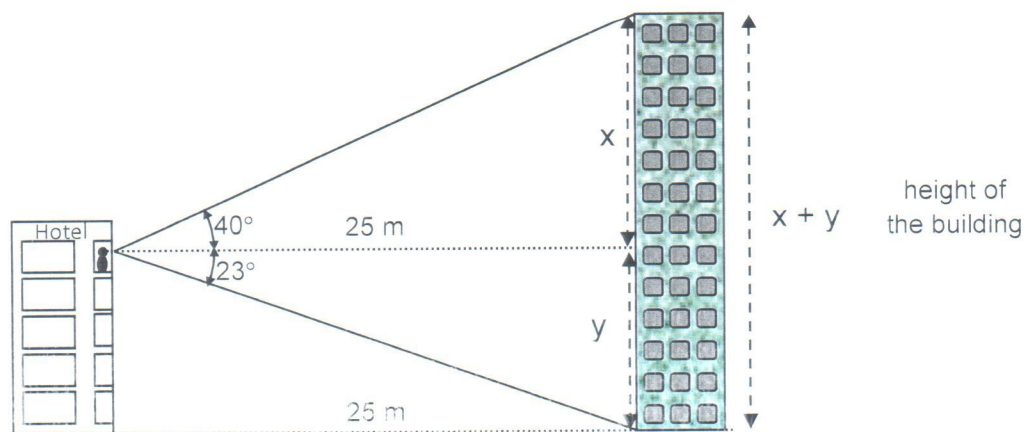
Simplify:

$$\text{Height of the lighthouse from sea level} = 15.31 \text{ km}$$

Therefore, the man is 15.31 km above sea level.

Example 4

A man on the fifth floor of a hotel observed that the angle of elevation from his place to the top of the opposite building is 40° , while the angle of depression to the ground of the same building is 23° . If the hotel and the building are 25 meters apart, how tall is the building?



What did you notice in this problem?

Great observation, if you say that the angle of elevation was not formed with the ground. Therefore it gives us an idea that the height of the building cannot be solved by only one step just like the previous examples.

Notice that we need to solve twice and add the heights ($x + y$) we derived using the two angles.

Let us first solve for the height of the building from the 5th floor up to its top (x). With that the following information are needed.

Given:

$$\theta = 40^\circ$$

horizontal distance between the hotel and building = 25 m

Use the formula and substitute the given in the formula

$$x = (\tan\theta)(\text{distance between the hotel and building})$$

$$x = (\tan 40^\circ) (25 \text{ m})$$

Simplify:

$$\mathbf{x = 20.98 \text{ m}}$$

Now we solve for the height from the fifth floor down to the ground.

With that the following information are needed.

Given:

$$\theta = 23^\circ$$

horizontal distance between the hotel and building = 25 m

Use the formula and substitute the given in the formula

$$y = (\tan\theta)(\text{distance between the hotel and building})$$

$$y = (\tan 23^\circ) (25 \text{ m})$$

Simplify:

$$\mathbf{y = 10.61 \text{ m}}$$

Now we already have the heights of x and y . We can now solve for the height of the building opposite the hotel by adding the two:

$$\text{Height of the building} = x + y$$

$$\text{Height of the building} = 20.98 \text{ m} + 10.61 \text{ m}$$

$$\text{Height of the building} = \mathbf{31.59 \text{ m}}$$

Therefore, the height of the building opposite the hotel is 31.59 m.

Solving for the Horizontal Distance

A simple formula just like the one used in finding the heights of objects can also be derived using the Tangent Ratio to facilitate speed and accuracy in solving.

Using the Tangent Ratio:

$$\tan \theta = \frac{\text{height of an object}}{\text{horizontal distance to object}}$$

Cross multiply:

$$(\tan \theta)(\text{horizontal distance to object}) = \text{height of an object}$$

Divide both sides by $\tan \theta$:

$$\frac{(\cancel{\tan \theta})(\text{horizontal distance to object})}{\cancel{\tan \theta}} = \frac{\text{height of an object}}{\tan \theta}$$

We simplify by interchanging the left and right sides of the equation:

$$\text{Horizontal distance to object} = \frac{\text{height of an object}}{\tan \theta}$$

Therefore, the simplest way to get the horizontal distance to object given the angle of elevation or depression we simply follow the formula:

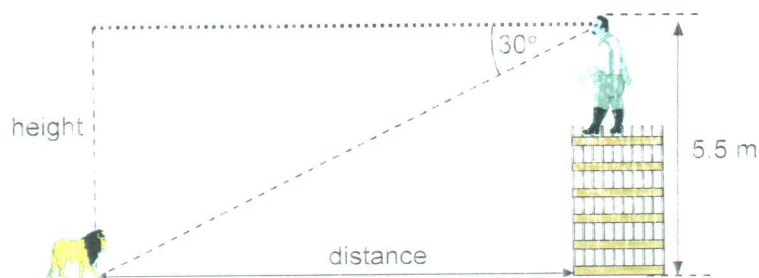
$$\text{Horizontal distance to object} = \frac{\text{height of an object}}{\tan \theta}$$

where: θ is the angle of elevation or depression

Example 5

From an elevated fence the zoo keeper sights a tiger on the ground at an angle of depression of 30° . If the zoo keeper and the fence make a total height of 5.5 m, how far is the lion from the zoo keeper?

Just like the previous examples, a sketch may no longer be necessary as long as you identify the given correctly and use the formula. But for this example, a sketch will be drawn.



Identify the given:

$$\theta = 30^\circ$$

Total height of the man and the fence = 5.5 m

Use the formula and substitute the given in the formula

$$\text{Horizontal distance to object} = \frac{\text{height of an object}}{\tan \theta}$$

$$\text{Horizontal distance to object} = \frac{5.5\text{m}}{\tan 30^\circ}$$

Simplify:

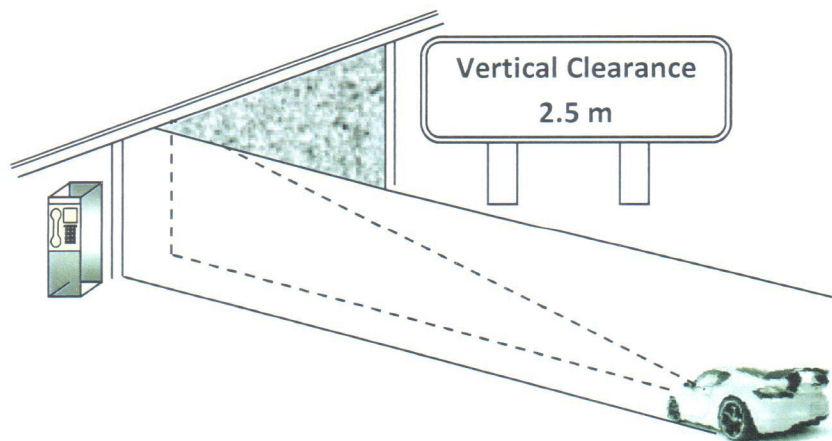
$$\text{Horizontal distance to object} = \frac{5.5\text{m}}{0.5774}$$

$$\text{Horizontal distance to object} = \mathbf{9.53 \text{ m}}$$

Therefore, the horizontal distance between the zoo keeper and the lion is 9.53 metres.

Example 6

A man runs out of petrol while on the road. When his car stopped, he saw a phone booth from a distance. He used his Clinometer and found out that the angle of elevation from the point where he is located to the top of the fly over is 20° . How far is he from the phone booth? (Use the information given in the figure).



Observe the figure above. From a sign board near the flyover, it says that the vertical clearance is 2.5 m. Therefore, we can say that it is also the approximate height of the flyover.

Notice that the phone booth can be found at the foot of the flyover. From there, we can say that the distance of the car from the flyover is also the same with its distance from the phone booth.

These analyses will lead us to the following valuable information:

Given:

$$\theta = 20^\circ$$
$$\text{height of the flyover} = 2.5 \text{ m}$$

Use the formula and substitute the given in the formula

$$\text{Horizontal distance from the flyover} = \frac{\text{height of the flyover}}{\tan \theta}$$

$$\text{Horizontal distance from the flyover} = \frac{2.5\text{m}}{\tan 20^\circ}$$

Simplify:

$$\text{Horizontal distance from the flyover} = \frac{2.5\text{m}}{0.3640}$$

$$\text{Horizontal distance from the flyover} = \mathbf{6.87\text{m}}$$

Therefore, the man is 6.87 m away from the phone booth.

If you have a handy Clinometer, this will help you calculate the approximate distance of an object from you given enough information you find around you.

Can you think of other instances wherein this knowledge you are learning will be applicable? Don't hesitate to share it with your friends and family.

Now be ready to answer the following practice exercise.

Enjoy learning!

NOW DO PRACTICE EXERCISE 3

**Practice Exercise 3**

1. Use the given problem to answer the questions that follow:

- a) The angle of elevation from a point on the ground to the top of a pole is 34° . How tall is the pole if the point is 12.5 meters away?

i. What are the given information?

ii. What is the appropriate formula to be used?

iii. Find the height of the pole.

- b) The angle of depression from the top of a 75-meter building to a car on the ground is 48° . How far is the car from the building?

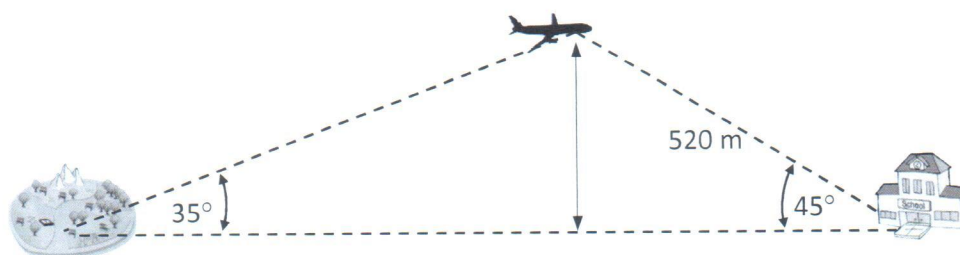
i. What are the given information?

ii. What is the appropriate formula to be used?

iii. Find is the distance between the car and the building?

2. Use the diagram to answer the questions that follow:

a)



i. What is the horizontal distance between the park and the airplane?

Answer: _____

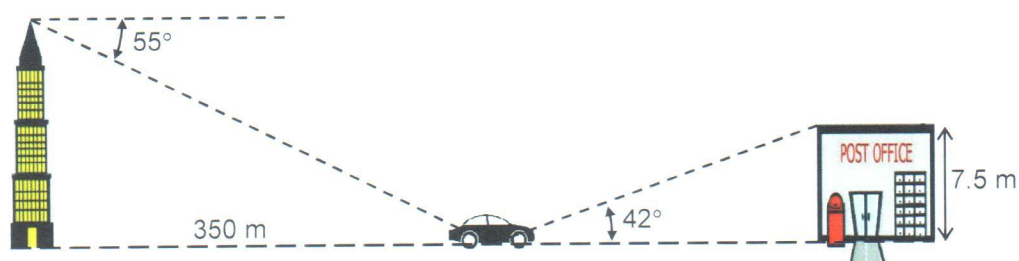
ii. Find the horizontal distance between the school and the airplane?

Answer: _____

iii. How far is the park from the school?

Answer: _____

b)



i. How high is the tower?

Answer: _____

ii. How far is the car from the Post Office?

Answer: _____

iii. What is the distance between the building and the Post Office?

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 4: Bearings

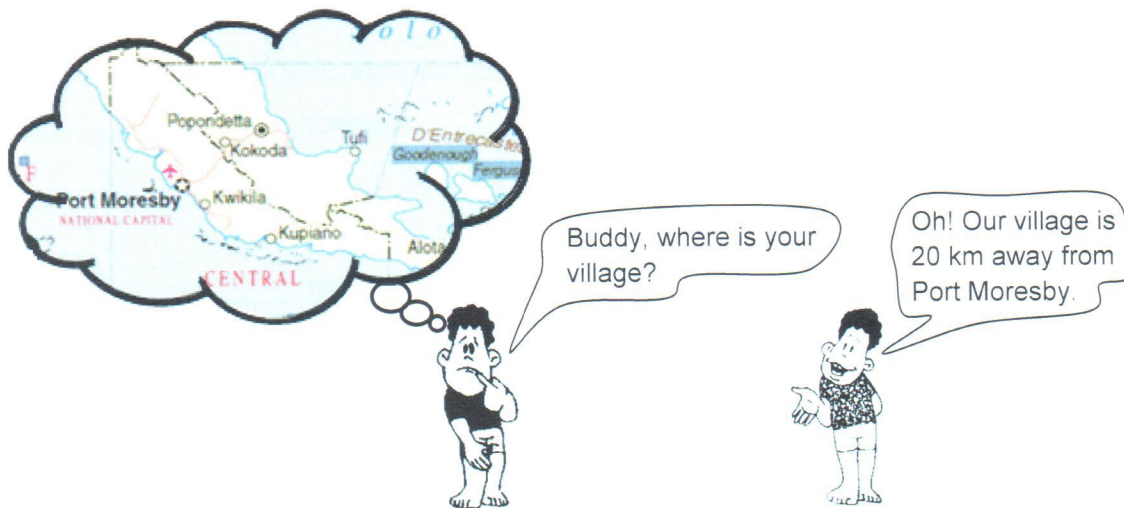


You learnt something about angles of elevation and depression in the previous lessons.



In this lesson, you will:

- define what a bearing is
- identify the purpose of using bearings
- draw diagrams involving bearings.



A comics strip above shows the conversation of two boys. Mike is asking John where his village is.

Why do you think Mike cannot figure out where John's village is?
If you are Mike, will you be able to tell John's village?

Definitely it will take a lot of guessing before one could figure out. Even if you have a map in front of you, you would not be able to locate it accurately. What do you think is missing in John's answer?

Brilliant, if you think of the direction!

If John would tell Mike that their village is 20 km North of Port Moresby, then the information is more accurate.

This situation emphasizes the need for a clear way of giving directions to locate accurately a particular place.



In navigation, it is very important to give an accurate detail of the location. A single small error in the location given may cause problems such as accidents among ships and airplanes.

Compass Direction

One of the basic ways of giving direction is by using the eight points of the compass:

N – North

E – East

S – South

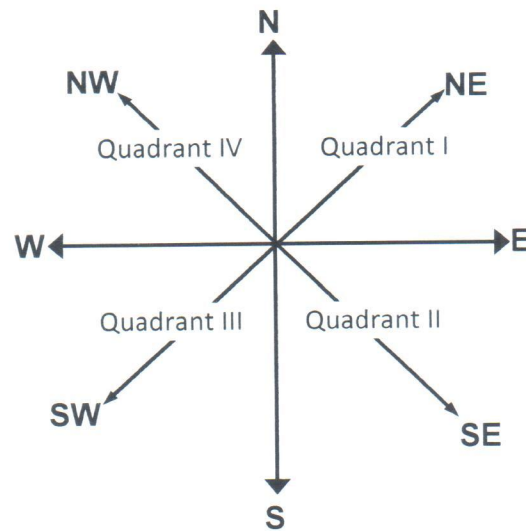
W – West

NE – North East

SE – South East

SW – South West

NW – North West



With the use of these directions, one can easily locate the direction of the place specifically with the aid of a compass and a map.

A more accurate and comprehensive way of giving direction is through the use of bearing.

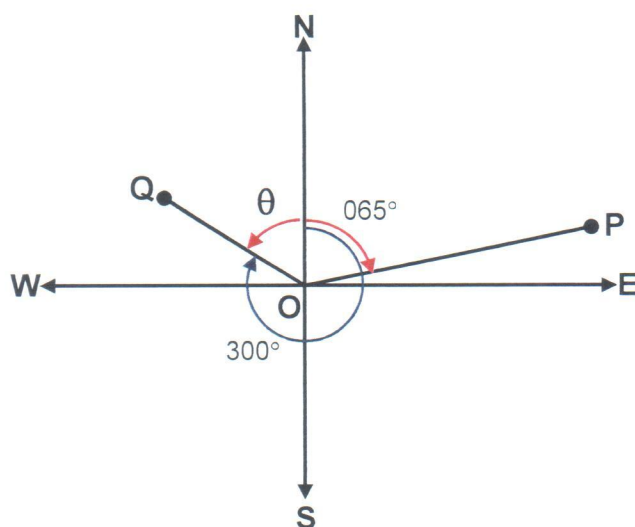
The **Bearing** to a point is the angle measured in degrees in a clockwise direction from the north line. In some references, it is referred to as the “**true bearing**”.

The true bearing is usually represented by 3 digits. A bearing of 055° mean that an angle of 55° measured in a clockwise direction from the north line is formed.

A **compass bearing** refers to an angle less than 90 degrees in between the north-south line and a line passing through a point from the west or east.

Using the previous example 055° , it will be named as $N55^\circ E$ using the compass bearing. The succeeding discussion will discuss how to transform true bearing to compass bearing and vice versa.

Observe the following figure below:



Example 1

What is the true bearing and the compass bearing of point **P** from **O**?

Solution:

In the figure the bearing of point **P** is 065° which is the number of degrees in the angle measured in a clockwise direction from the north-line to the line joining the centre of the compass at **O** with the point **P** (i.e. **OP**).

Since point **P** is in quadrant 1, the angle is the same. The only difference is the way it is written. Its equivalent compass bearing is $N65^\circ E$ (read as 65 degrees Northeast).

Example 2

What is the bearing of **Q** from **O**?

Solution:

The bearing of point **Q** is 300° which is the number of degrees in the angle measured in a clockwise direction from the north-line to the line joining the centre of the compass at **O** with the point **Q**.

Since point **Q** is in quadrant 2, angle is not the same with the true bearing just like in example 1. Its equivalent compass bearing is derived by subtracting 300° from 360° . So we have $360^\circ - 300^\circ = 60^\circ$.

Why subtract 300 degrees?

Notice that 1 revolution is equal to 360° and if you add the value of θ with 300° their sum is equivalent to 1 revolution. Also notice that the compass bearing as defined is an angle less than 90 degrees in between the north-south line and a line passing through a point from the west or east. Since **Q** is in the fourth quadrant, therefore it is on the west of **O**.

Therefore its compass bearing is $N60^{\circ}W$ (read as 60 degrees Northwest).

REMEMBER! The bearing of a point is the number of degrees in the angle measured in a clockwise direction from the north line to the line joining the center of the compass with the point while the compass bearing is an acute angle formed by the north-south line and a point in any of the fourth quadrant.

Example 3

a) Give the equivalent compass bearing of the following:

i. 080°

ii. 200°

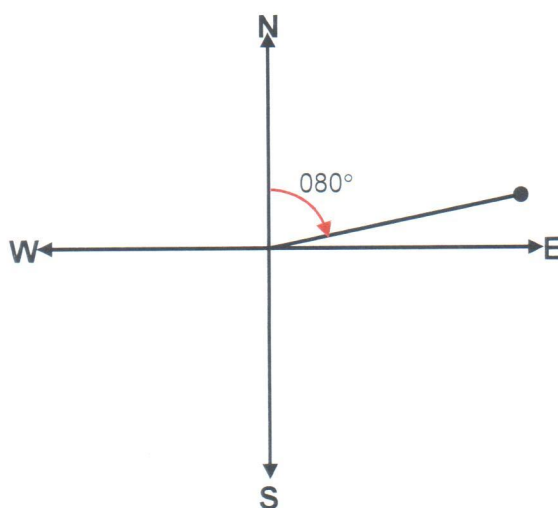
b) Give the equivalent true bearing of the following:

i. $S30^{\circ}E$

ii. $N40^{\circ}W$

Solution:

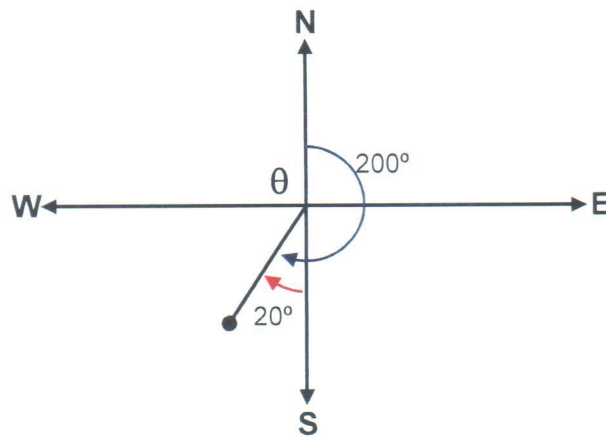
a) i. 080° is found in the first quadrant.



If the bearing is less than 90° , then it is found on the first quadrant and it is North East (NE) from the reference point, which is the North line.

Therefore, 080° in compass bearing is expressed as $N80^{\circ}E$.

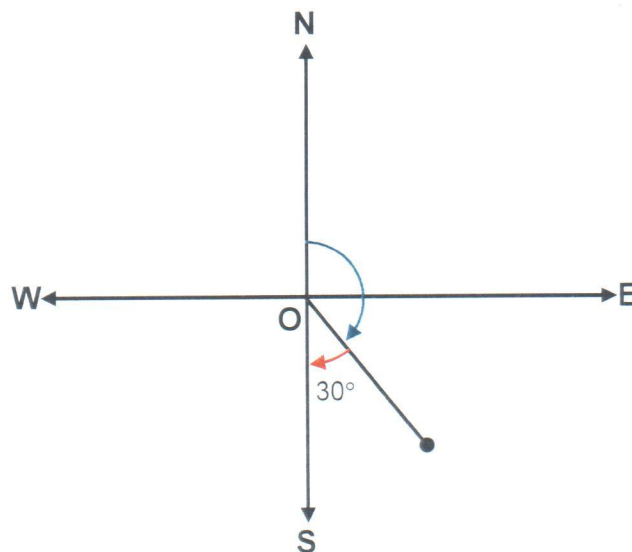
- ii. 200° is found in the third quadrant.



If the bearing is more than 180° but less than 270° , it is found on quadrant III and it is South West (SW) from the reference point which is the north line. However, we cannot write $S200^\circ W$ because compass bearing only uses measures less than 90° . In that case, we subtract 180° from 200° ($200^\circ - 180^\circ = 20^\circ$). Why 180° ? Referring to our analysis in examples 1 and 2, we will realize that the amount of rotation from the north line, 200° passes through the south line (180°), so the difference of 200° and 180° is the angle less than 90° from the south line as shown in the illustration on page 39. Since the angle is found in the third quadrant, we say that the bearing 20° is in South West.

Therefore, 200° in compass bearing is expressed as $S20^\circ W$.

- b) i. To express $S30^\circ E$ to its equivalent true bearing, sketch and locate the given compass bearing.

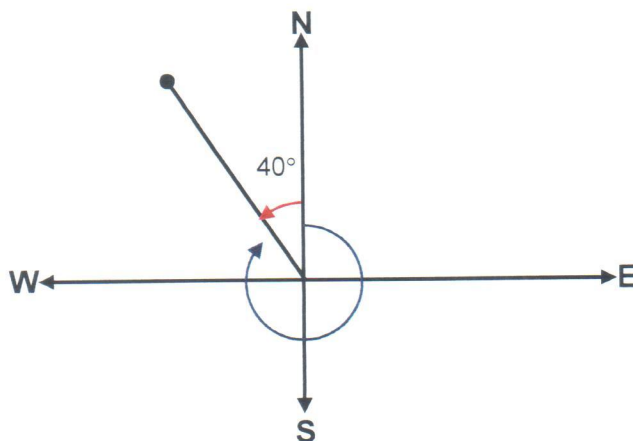


Since, $S30^\circ E$ is in quadrant II as shown in the illustration above, we can get the true bearing by subtracting 30° from 180° ($180^\circ - 30^\circ = 150^\circ$).

Therefore, $S30^\circ E$ in true bearing is expressed as 150° .

- ii. Expressing $N40^\circ W$ to its equivalent true bearing, it's easier to analyse using an illustration.

First, locate the given compass bearing.

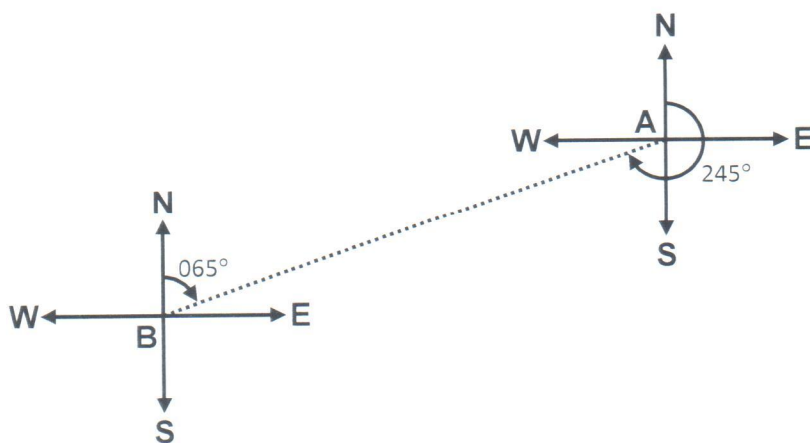


Since $N40^\circ W$ is in quadrant IV as shown in the illustration in the previous page, we can derive the true bearing by subtracting 40° from 360° ($360^\circ - 40^\circ = 320^\circ$).

Therefore, $N40^\circ W$ in true bearing is expressed as 320° .

A bearing is used to represent the direction of one point relative to another point.

In the figure below, the bearing of A from B is 065° . The bearing of B from A is 245° . Note that all bearings are three figures and they are measured in a horizontal plane.

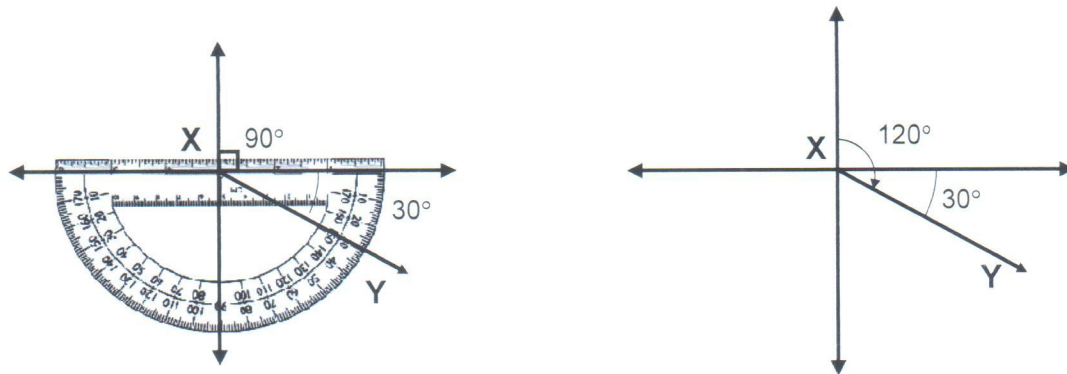


In navigation, commonly the true bearing is used because it gives a more accurate and simpler way of representing it without dealing with a lot of directions. However, we cannot dispense the importance and use of the compass bearing.

Any of the two could be used depending on the need and where you will use it. Aircraft navigation may require the use of true bearing while a simple map to be followed in a camping may require the use of compass bearing. That is why it is important that you will be able to read bearing and translate a true bearing to its equivalent compass bearing and vice versa.

Example 4

Draw the bearing of **Y** from **X** which is 120° and give its equivalent compass bearing.



Using the protractor, measure 30° on the second quadrant and adding this to the measure of the first quadrant, we get, $90^\circ + 30^\circ = 120^\circ$.

The equivalent compass bearing of 120° is $S60^\circ E$.

NOW DO PRACTICE EXERCISE 4

**Practice Exercise 4**

1. Draw diagrams of the following bearings if the bearing from **Y** to **X** is:

a) $N45^{\circ}E$

c) $S25^{\circ}E$

b) $S20^{\circ}W$

d) $N50^{\circ}W$

2. Draw diagrams of the following bearings if the bearing from **B** to **A** is:

a) 220°

c) 025°

b) 340°

d) 155°

3. Write the equivalent compass bearing of the following:

a) 23° _____

b) 188° _____

c) 245° _____

d) 340° _____

e) 122° _____

4. Write the equivalent true bearing of the following:

a) $N15^\circ W$ _____

b) $S20^\circ W$ _____

c) $N18^\circ W$ _____

d) $S 27^\circ E$ _____

e) $N24^\circ E$ _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.
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Lesson 5: Problems on Bearings



You learnt about true bearing and compass bearing in Lesson 4.

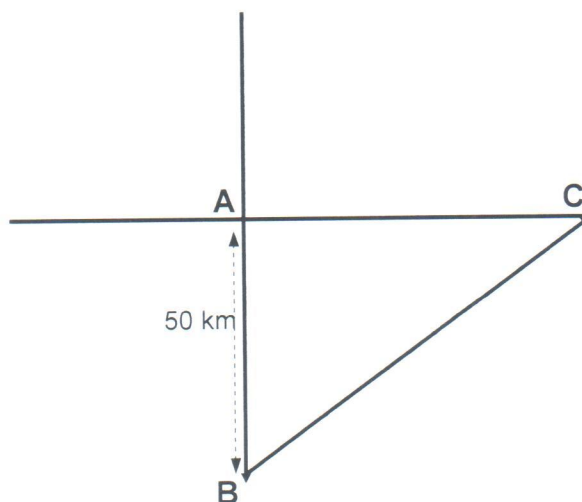


In this lesson, you will:

- solve problems on bearing using sine, cosine and tangent ratios.

Knowledge on bearing is very useful especially in navigation. This topic will help you a lot in solving real-life problems. More so, if you are dreaming to become a pilot or a sea fearer someday, you will find this Lesson very helpful and interesting.

A ship sails south for 50 kilometres from a point A to a point B, and then sails on a bearing of N45°E to a point C which is due east of its starting point. How far is the ship from its starting point?



Notice that right triangle CAB with a right angle A is formed in the figure. An acute angle ABC measuring 45 degrees is also formed. From this, we will be able to use the Tangent Ratio to solve for AC.

Form the tangent ratio:

$$\tan 45^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{50}$$

Cross multiply:

$$(\tan 45) (50\text{km}) = AC$$

Simplify (use trigonometric table):

$$(1)(50\text{km}) = AC$$

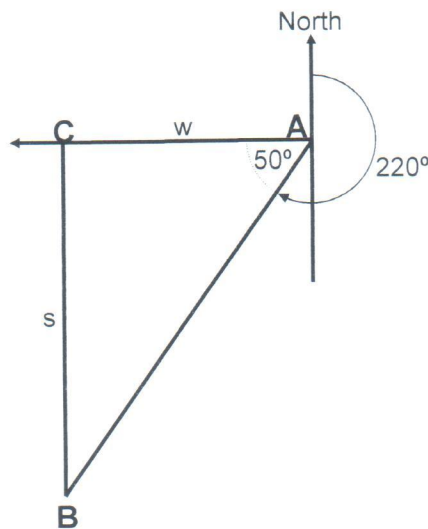
$$AC = 50 \text{ km}$$

Therefore, the ship is 50 km away from its starting point.

Example 2

A boat sails from port A on a bearing of 220° . After sailing for 100 km, it reaches port B.

- How far South (s) is port B from port A?
- How far West (w) is port B from A?
- What is the equivalent compass bearing of A from B?



Solution:

- In the figure we formed the right triangle ACB where angle C is a right. Since the acute angle CAB is in the third quadrant, we subtract the given 220° from 270° to get the measure of angle CAB. That is, $270^\circ - 220^\circ = 50^\circ$.

We will use this acute angle to solve for the distance between C and B.

Form the sine ratio:
$$\sin 50^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{s}{100}$$

Cross multiply:
$$(\sin 50)(100\text{km}) = s$$

Simplify (use trigonometric table):
$$(0.766)(100\text{km}) = s$$

$$s = 76.6 \text{ km}$$

Therefore, the boat is 76.6 km south from A.

- To find the distance between A and C, we use the cosine ratio.

Form the cosine ratio:
$$\cos 50 = \text{adj} / \text{hyp} = w / 100$$

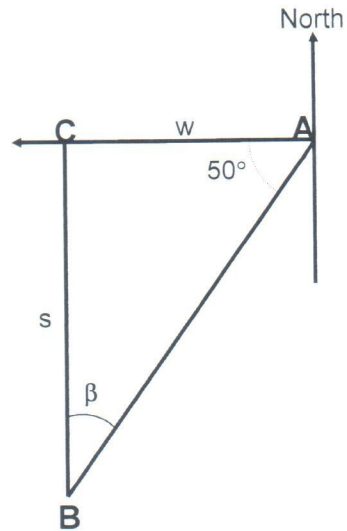
Cross multiply:
$$(\cos 50)(100\text{km}) = w$$

Simplify (use trigonometric table):
$$(0.6428)(100\text{km}) = w$$

$$w = 64.28 \text{ km}$$

Therefore, the boat is 64.28 km west from A.

- c) The true bearing of A from B is shown as β on the diagram below.



We know that the sum of all angles in a triangle is 180° .

We now say that $\beta = 180^\circ - 90^\circ - 50^\circ = 40^\circ$.

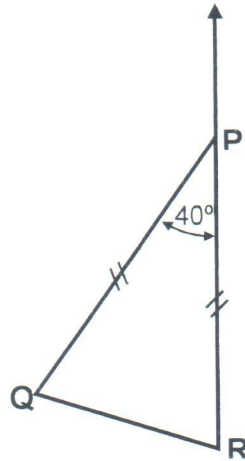
Therefore the true bearing of A from B is 040.

NOW DO PRACTICE EXERCISE 5



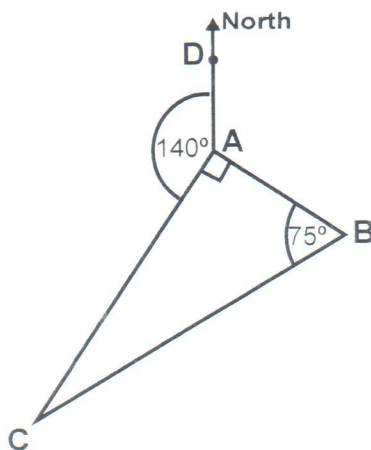
Practice Exercise 5

1. **P**, **Q** and **R** are three points on level ground with **P** due north of **R**. Angle $\text{QPR} = 40^\circ$ and $\text{PQ} = \text{PR}$. Calculate the true bearing of:



- (a) **Q** from **P** _____
- (b) **P** from **Q** _____
- (c) **Q** from **R** _____
- (d) **R** from **P** _____

2. Points **A**, **B**, **C** and **D** lie on level ground. The point **D** is due north of **A**. $\angle \text{DAC} = 140^\circ$, $\angle \text{CAB} = 90^\circ$ and $\angle \text{ABC} = 75^\circ$. Find the bearing of:



- (a) **A** from **C** _____
- (b) **A** from **B** _____
- (c) **C** from **A** _____
- (d) **C** from **B** _____

3. Joshua walks 3 km due east from a point **P** to **Q**. From **Q**, he walks a further distance of 5 km on a bearing of 054 to a point **R**. Calculate the total distance travelled by Joshua from point **P** to point **Q**.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 6: Mixed Problems



You have learnt about bearings and its types and solved problems involving heights and distances in the previous lessons.



In this lesson you will:

- solve mixed problems involving height, distances and bearing.

In some cases, you will need more than one skill to deal with a certain situation or problem.

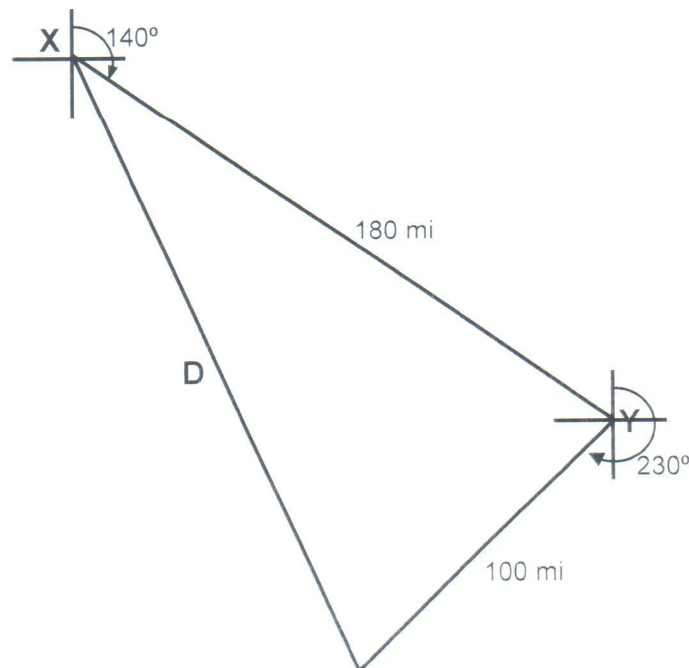
Imagine, after you have measured the bearing of a person from the top of a building, what will you do with the measurement you just derived?

Definitely, measuring the bearing will be followed by determining the distance of the person from the building. This may require another skill which will involve solving for distances and heights.

In this lesson, we will focus on the application of the previous skills mastered wherein a combination of different skills are involve.

Example 1

An airplane flies 180 miles from point **X** at a bearing of 140 degrees, and then turns and flies at a bearing of 230 degrees for 100 miles. How far is the plane from point **X**?



Solution:

Analysing the figure, we can prove that the triangle formed is a right angled. How?

Consider the bearing of **Y** from **X** ($140^\circ - 90^\circ = 50^\circ$) and the bearing of **X** from **Y** ($270^\circ - 230^\circ = 40^\circ$), they add up to 90° .

Through the alternate interior angle theorem discussed in Unit 5, we can clearly see in the figure that adding up the two angles makes a right angle.

To solve for **D**, use the Pythagorean Theorem where **D** is the hypotenuse and 180 miles and 100 miles are the legs.

$$D = \sqrt{180^2 + 100^2} = \sqrt{32400 + 10000} = \sqrt{42400} = 205.91 \text{ miles.}$$

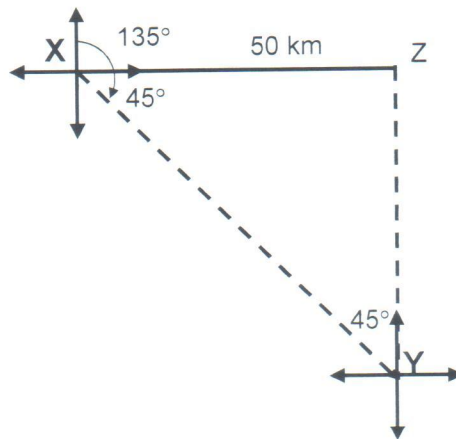
Therefore, the plane is 205.91 miles away from the starting point X.

Example 2

A man walks 50 km due East which is North of **Y**. How far is the man from point **Y** if the bearing of **Y** from **X** is 135° ?

Solution

The problem can be analysed using the figure below.



Since the man (**Z**) is now standing East of **X** and North of **Y**, we can analyse that a right angle is formed. In addition, the bearing 135° gives a 45° angle of the triangle ($135^\circ - 90^\circ = 45^\circ$). Recall our lesson on special right triangles in the previous units that a 45° - 90° - 45° is an isosceles right triangle. If the triangle is an isosceles, then we can conclude that the distances between **XZ** and **YZ** are equal in measure. So by analysis alone, we can say that the measure of **YZ** is also 50 km.

Or the tangent ratio can be used. If $\tan 45^\circ = \frac{YZ}{50\text{km}}$, then $YZ = (\tan 45^\circ)(50 \text{ km})$. But $\tan 45^\circ$ is equal to 1, so we have $(1)(50\text{km}) = 50 \text{ km}$.

Therefore, the man is 50 km away from point Y.

NOW DO PRACTICE EXERCISE 6



Practice Exercise 6

Solve the following problems. Use the spaces for your working out.

1. A hiker found that he has to complete a five – mile course which ends S30°E of the starting point. How far south is he from his starting point?
2. A ship travels N50°E until it is due North of a port 10 nautical miles due East of the starting point. °How far did the ship travel?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

TOPIC 1: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- Indirect Measurement - is a technique that uses proportions to find a measurement when direct measurement is not possible.
- The angle of elevation is the angle formed by the horizontal and the line of sight above it.
- The angle of depression is the angle formed by the horizontal and the line of sight below it.
- A clinometer is a tool or an instrument used to measure angles of elevation and depression.
- The height of an object is derived by the product of $\tan \theta$ and horizontal distance to object where θ is the angle of elevation or depression.
- horizontal distance to object = $\frac{\text{height of an object}}{\tan \theta}$
where: θ is the angle of elevation or depression
- The Bearing to a point is the angle measured in degrees in a clockwise direction from the north line. In some references it is referred to as the "true bearing".
- A compass bearing refers to an angle less than 90 degrees in between the north-south line and a line passing through a point from the west or east.

REVISE LESSON 1 TO 6. THEN DO TOPIC TEST 1 IN ASSIGNMENT BOOK 6.

ANSWERS TO PRACTICE EXERCISES 1 - 6

Practice Exercise 1

1.
 - a) Angle of Elevation
 - b) NA (not applicable)
 - c) Angle of Depression
 - d) NA (not applicable)
 - e) Angle of Elevation
 - f) Angle of Depression
 2.
 - a) length of wire BC = 6.73 m
 - b) length of wire AE = 12.45 m
 - c) height of that wire BC reach (BD) = 4.50 m
 - d) height of the flagpole (AD) = 9.53 m
 - e) width of the flag (AB) = 5.03 m
 3.
 - a) 22.5 m
 - b) 137.37 m
-

Practice Exercise 2

1.
 - a) Angle of Elevation = 20°
 - b) Angle of Elevation = 40°
 - c) Angle of Depression = 50°
 - d) Angle of Depression = 30°
 - e) Angle of Elevation = 40°
-

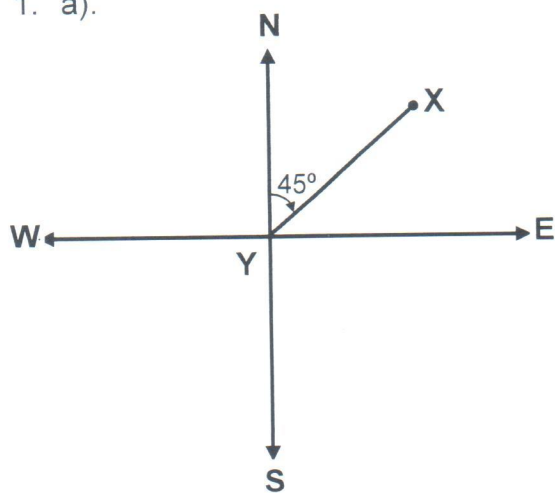
Practice Exercise 3

1.
 - a)
 - i. Given Information: angle of elevation, 34° ; distance between top of the pole and ground, 12.5 m; unknown height of pole.
 - ii. The Sine Ratio: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
 - iii. $h = 6.71 \text{ m}$
 - b)
 - i. Given Information: angle of depression, 48° ; height of building, 75 m; unknown distance between car and the building.
 - ii. The Tangent Ratio: $\tan \theta = \frac{\text{opp}}{\text{adj}}$
 - iii. $d = 67.53 \text{ m}$

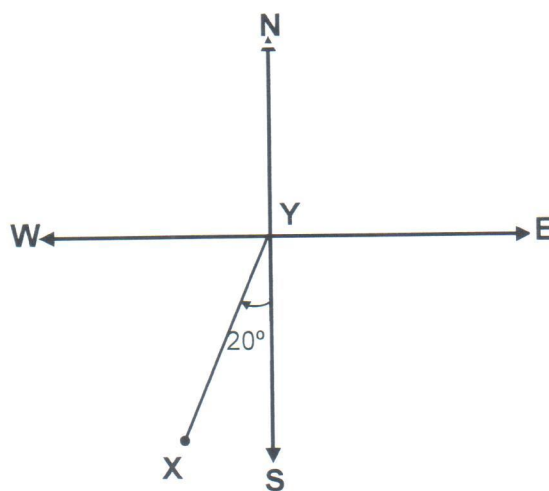
2. a). i. 742.64 m
ii. 520 m
iii. 1262.64 m
- b) i. 499.85 m
ii. 8.33 m
iii. 508.18 m

Practice Exercise 4

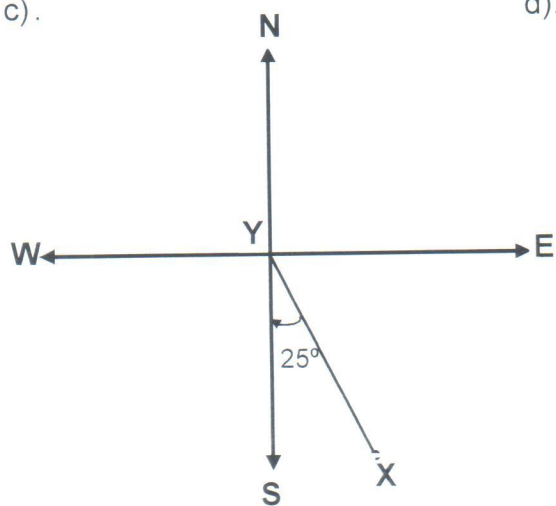
1. a).



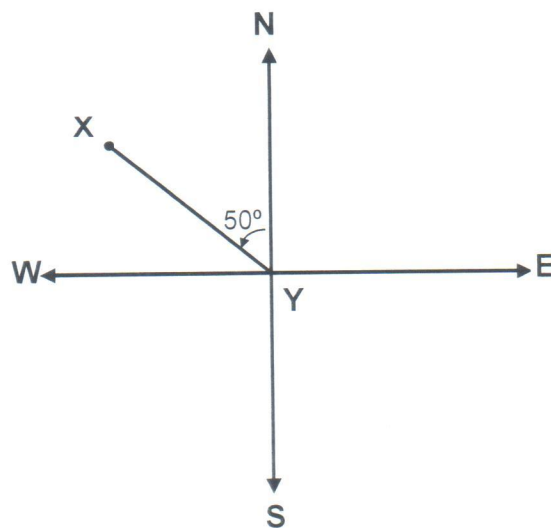
b).



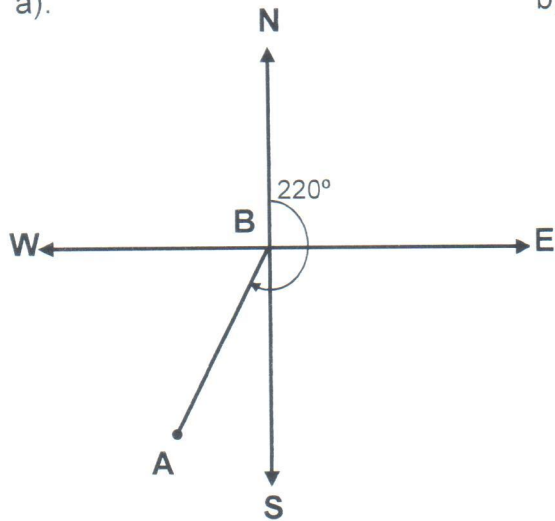
c).



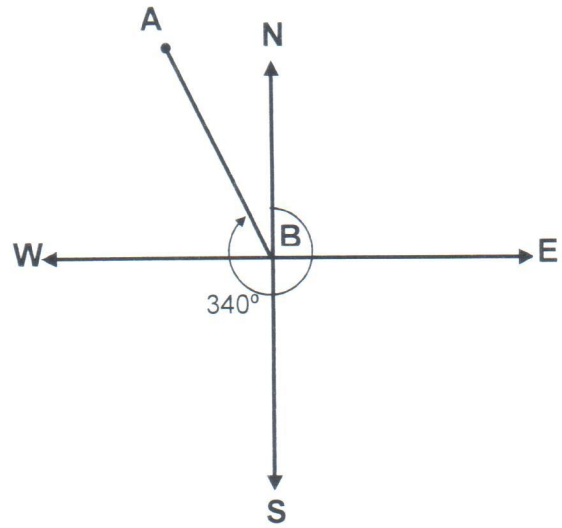
d).



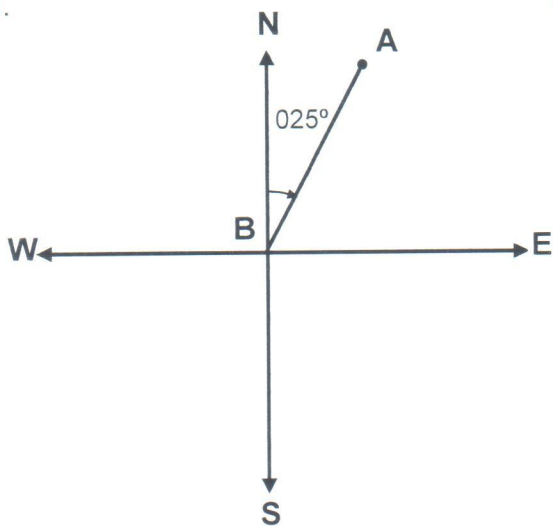
2. a).



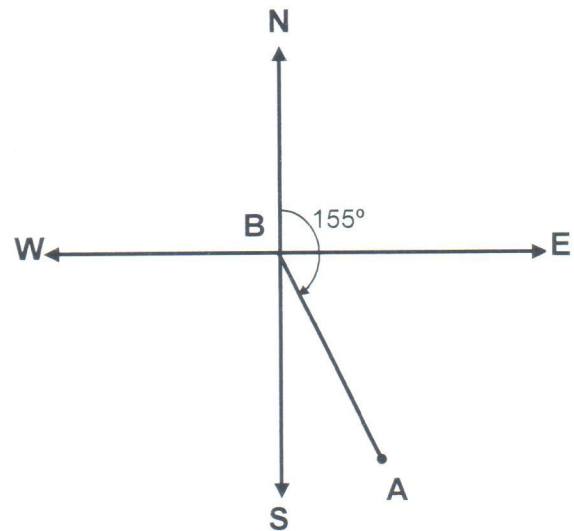
b).



c).



d).



3. a) $N23^{\circ}E$
 b) $S8^{\circ}W$
 c) $S65^{\circ}W$
 d) $N20^{\circ}W$
 e) $S58^{\circ}E$

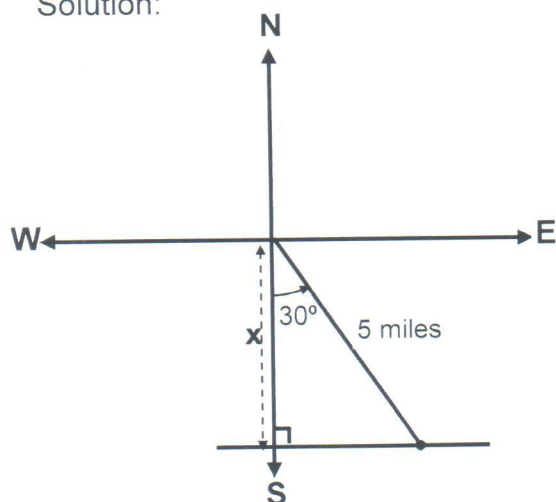
4. a) 345°
 b) 200°
 c) 342°
 d) 153°
 e) 024°

Practice Exercise 5

1. a) 220° b) 040° c) 290° d) 180°
 2. a) 040° b) 220° c) 235°
 3. $3 \text{ km} + 5 \text{ km} = 8 \text{ km}$

Practice Exercise 6

1. Solution:



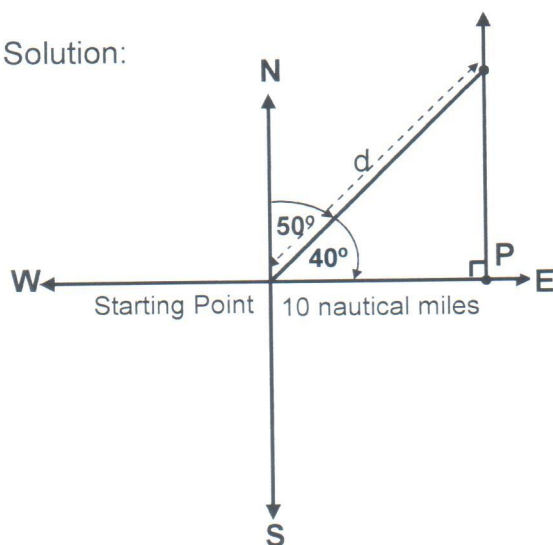
$$\cos 30^\circ = \frac{x}{5}$$

$$5 \times \cos 30^\circ = \frac{x}{5} \times 5$$

$$x = 5 \times \cos 30^\circ$$

$$x = 4.33 \text{ miles}$$

2. Solution:



$$\cos 40^\circ = \frac{10}{d}$$

$$x = \frac{10}{\cos 40^\circ}$$

$$x = 13.05 \text{ nautical miles}$$

END OF TOPIC 1

TOPIC 2

SCALE AND SIMILARITY

Lesson 7:	Scale Models
Lesson 8:	Plans
Lesson 9:	Maps
Lesson 10:	Enlargement and Similar Shapes
Lesson 11:	Area of Similar Shapes

TOPIC 2: SCALE AND SIMILARITY

Introduction



Have you seen miniature objects such as cars, houses and gadgets?

Maybe you wonder why these objects resemble the real ones and have all the necessary details found in the real objects, however they are so small.



In this topic, we will discuss ways of enlarging or reducing the sizes of objects without altering their details.

Scale models, plans and maps will also be discussed in this topic.

You will explore ways on how to create a plan, read maps and use scaling to accurately indicate certain locations in the maps or plans.

The skills you will learn in this topic will be very helpful if you intend to pursue courses which relates to architecture, engineering, technical and vocational studies which involve production of crafts.

Lesson 7: Scale Models



You have learnt about models in your lower Mathematics.



In this lesson, you will:

- define what a scale model is
- workout problems involving scale models.

A **scale model** is a representation or copy of an object which shows accurate relationship between all important aspects of the object. The model can be larger or smaller but its size is in fixed proportion with the actual size of the object. The first picture shows a car and the second one is toy car scaled from the original one.



Notice that the two are similar in all aspect. Each physical detail of the real car can be found on the toy car. That makes the toy car a good example of a scale model.

Let us have another example. A kundu measures 90 cm long and its diameter is 15 cm.



A scale model smaller than the original one was made. Its length is 30 cm and its diameter of 5 cm.

Observe the following ratios formed using the measures of the actual and model kundus.

Length of the model kundu : Length of the actual kundu

$$30 \text{ cm} : 90 \text{ cm} = 1 : 3$$

Diameter of the model kundu : Diameter of the actual kundu :

$$5 \text{ cm} : 15 \text{ cm} = 1:3$$

The scale factor for the smaller model is 1:3 or $\frac{1}{3}$.

This means that the model is $\frac{1}{3}$ of the size of the actual model. We can also say that the actual kundu is three times the size of the model.

Now observe the pictures below.

Can we say that the second box is a bigger model of the first?



They may look the same in terms of their colors and design (ribbon), but observe them carefully.

Brilliant observation, if you say that the two are not similar. Why?

The faces of the first box are all squares while the second one has squares and rectangles.

We can verify if the second is not really a scaled model of the first by forming ratios on each corresponding sides.

$$\text{Length of the original : Length of the model} \quad 10 \text{ cm} : 35 \text{ cm} = 2:7$$

$$\text{Width of the original : Width of the model} \quad 10 \text{ cm} : 20 \text{ cm} = 2:4$$

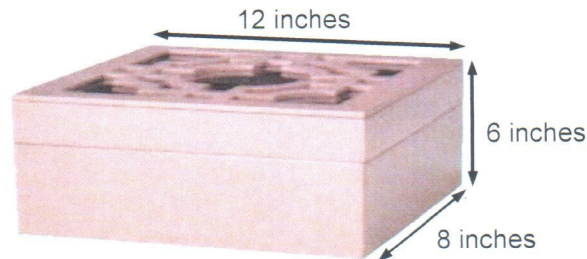
$$\text{Height of the original : Height of the model} \quad 10 \text{ cm} : 15 \text{ cm} = 2:3$$

Since $\frac{2}{7} \neq \frac{2}{4} \neq \frac{2}{3}$, no proportion was formed, we conclude that the two boxes above are not similar and the second one is not a model of the original box.

Similar objects are two or more objects whose corresponding parts (sides) form a proportion.

Example 1

A group of students were asked by their teacher to create (A) larger and (B) smaller models of the wooden box she gave them for the school exhibit. The teacher instructed the students to create a smaller box half the size of the original, while the bigger one must be used as a scale factor of 1:7. What are the dimensions of the model boxes?



Solution:

- (A) Since the bigger one requires a scale factor of 1:7, it means that the original box is only $\frac{1}{7}$ of the model. Or the model is 7 times the size of the original.

Length:

Length of the original box : Length of the model = 1:7

$$12 \text{ inches} : L = 1 : 7$$

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 extremes means

Recall! In a proportion, the product of the MEANS is EQUAL to the product of the EXTREMES.

So we say, $(L)(1) = (12 \text{ inches})(7)$

$$L = 84 \text{ inches}$$

Width :

Width of the original box : Width of the model = 1:7

$$8 \text{ inches} : W = 1 : 7$$

Getting the product of the means and extremes,

$$(W)(1) = (8 \text{ inches})(7)$$

$$W = 56 \text{ inches}$$

Height:

height of the original box : height of the model = 1:7

$$6 \text{ inches} : H = 1 : 7$$

Getting the product of the means and extremes,

$$(H)(1) = (6 \text{ inches})(7)$$

$$H = 42 \text{ inches}$$

Therefore, the dimensions of the bigger model box are 84 inches (length), 56 inches (width) and 42 inches (height).

- (B) Since the smaller box must be half the size of the original, the scale factor to be used must be 1:2.

Length:

Length of the model box : Length of the original box = 1:2

$$L : 12 \text{ inches} = 1 : 2$$

Getting the product of the means and extremes,

$$(L)(2) = (12 \text{ inches})(1)$$

Divide both sides by 2

$$\mathbf{L = 6 \text{ inches}}$$

Width:

Width of the model box : Width of the original box = 1:2

$$W : 8 \text{ inches} = 1 : 2$$

Getting the product of the means and extremes,

$$(W)(2) = (8 \text{ inches})(1)$$

Divide both sides by 2

$$\mathbf{W = 4 \text{ inches}}$$

Height:

Height of the model box : Height of the original box = 1:2

$$H : 6 \text{ inches} = 1 : 2$$

Getting the product of the means and extremes,

$$(H)(2) = (6 \text{ inches})(1)$$

Divide both sides by 2

$$\mathbf{H = 3 \text{ inches}}$$

Therefore, the dimensions of the smaller model box are 6 inches (length), 4 inches (width) and 3 inches (height).

Example 2

Use the picture with its given dimensions and scale factor to answer the following questions:



Scale: 1 cm : 20 inches

- What is the actual length of the bag?
- What is the actual width of the bag?
- What is the actual height of the box?

Solution:

- Since the scale factor of the drawing to the actual bag is 1 cm : 20 inches or 1:20

We form the proportion:

$$1 \text{ cm} : 20 \text{ inches} = 5 \text{ cm} : \text{length}$$

Get the product of the means and the extremes:

$$\text{Length}(1 \text{ cm}) = (20 \text{ inches})(5 \text{ cm})$$

Divide both sides by 1 cm:

$$\frac{\text{Length}(\cancel{1 \text{ cm}})}{\cancel{1 \text{ cm}}} = \frac{(20 \text{ inches})(\cancel{5 \text{ cm}})}{\cancel{1 \text{ cm}}}$$

Simplify:

$$\text{length} = 100 \text{ inches}$$

- We form the proportion:

$$1 \text{ cm} : 20 \text{ inches} = 1 \text{ cm} : \text{width}$$

Get the product of the means and the extremes:

$$\text{Width}(1 \text{ cm}) = (20 \text{ inches})(1 \text{ cm})$$

Divide both sides by 1 cm:

$$\frac{\text{Width}(\cancel{1 \text{ cm}})}{\cancel{1 \text{ cm}}} = \frac{(20 \text{ inches})(\cancel{1 \text{ cm}})}{\cancel{1 \text{ cm}}}$$

Simplify:

$$\text{Width} = 20 \text{ inches}$$

- We form the proportion:

$$1 \text{ cm} : 20 \text{ inches} = 3 \text{ cm} : \text{height}$$

Get the product of the means and the extremes:

$$\text{height}(1 \text{ cm}) = (20 \text{ inches})(3 \text{ cm})$$

Divide both sides by 1 cm:

$$\frac{\text{Height}(\cancel{1 \text{ cm}})}{\cancel{1 \text{ cm}}} = \frac{(20 \text{ inches})(\cancel{3 \text{ cm}})}{\cancel{1 \text{ cm}}}$$

Simplify:

$$\text{Height} = 60 \text{ inches}$$

Example 3

In a museum a model dinosaur stands 6.5 ft tall. If the model is a smaller version of the real dinosaur in a scale factor of 1:15, what is the actual height of the dinosaur?

Solution:

Height of the model dinosaur: Height of the actual dinosaur = 2:15

Substitute the given values:

6.5 ft: Height of the actual dinosaur = 2:15

Cross multiply:

$$(2) (\text{Height of the actual dinosaur}) = (15) (6.5 \text{ ft})$$

Divide both sides by 2:

$$\frac{(2) (\text{Height of the actual dinosaur})}{2} = \frac{(15) (6.5 \text{ ft})}{2}$$

Simplify:

$$\text{Height of the actual dinosaur} = 48.75 \text{ ft}$$

Therefore the actual dinosaur lived stood at 48.75 ft.

NOW DO PRACTICE EXERCISE 7



Practice Exercise 7

Solve the following:

1. A 6 inches by 15 inches photo is to be enlarged with a scale factor of 1: 8. What are the dimensions of the bigger photo?
2. A statue stands at 12 ft. A smaller replica (model) of it is to be displayed in a museum. The actual statue is five times the height of its replica.
 - a) What scale factor was used to make the replica?
 - b) What is the height of the replica?
 - c) If a scale factor of 2:3 is to be used, what could be the height of the model?
3. In vicinity map drawn, every centimeter is a representation of 5 meters in actual setting.
 - a) How far is the school from the market if the map shows a distance of 19.25 cm?
 - b) If the actual bridge is 38 meters long, how long will it be represented in the map?

c) If a soccer field measures 15 cm by 22 cm, what are their actual measures?

d) What is the area of the actual field?

4. An airline wishes to make a model of their new aircraft to be sold as a toy during their flights. The proposed model would be made in a scale factor of 1 : 900.

a) What is the actual length of the plane in meters if the toy measures 30 cm?

b) If a helicopter is also to be modeled using the given scale factor, what is the length of its model if the helicopter's length is 13 meters?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.
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Lesson 8: Plans



You learnt about scale model in the previous lesson.



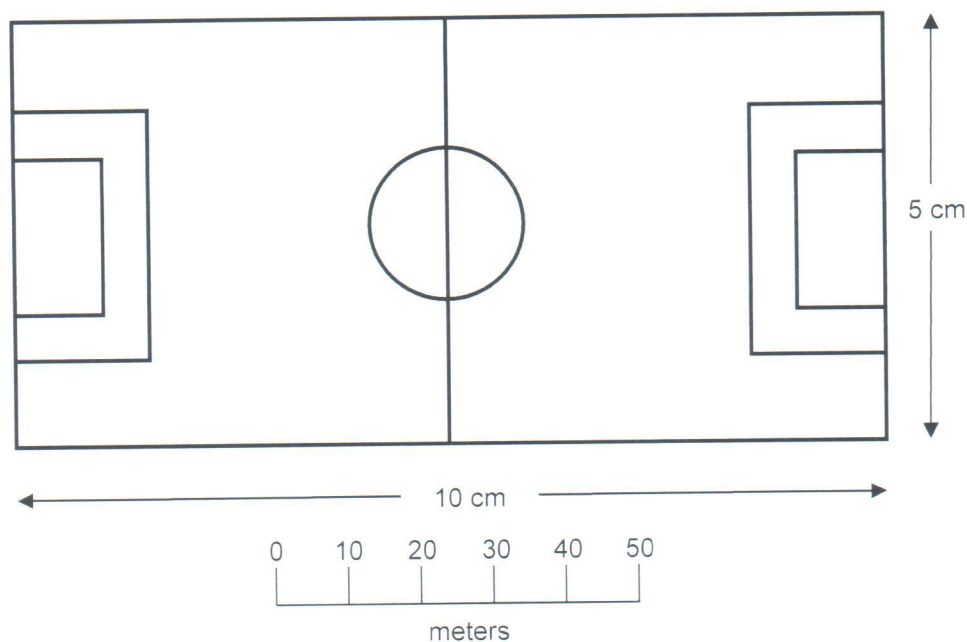
In this lesson, you will:

- define and explain what a plan is
- use scales in a plan.

What is a plan?

A plan is a line drawing of a land, a building or any object drawn to a scale. A plan is a complete illustration of the important parts of the original object or structure which includes the measurement, the shape, the design and other specific details.

The drawing below is a plan for a soccer field.



The plan shows specific details such as the center line, the four sides and the measurement of the field. Below is a bar scale used.

Drawings on a plan are drawn using a scale. Standard fractions are used in relation to the actual size of the object being drawn and the dimensions used on the diagram. In the plan illustrated above, a bar scale was used. It shows that 1 cm on the plan represents 10 meters on the ground.

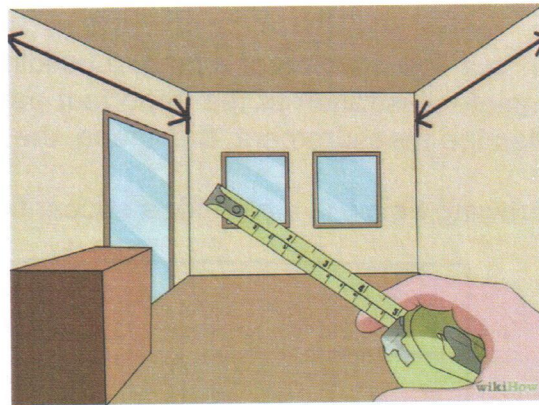
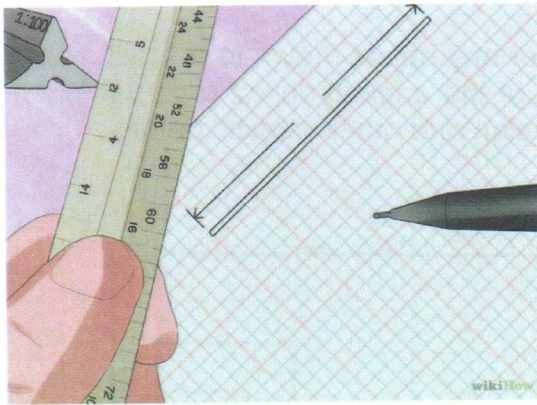
Architectural drawings are done in scales that are smaller than the real size. Floor plans are commonly produced at $1/50$ th (1 to 50) or $1/100$ th (1 to 100) of their actual size. Site plans are often drawn at $1/200$ th (1 to 200) or even $1/500$ th (1 to 500) of actual size.

A scale of 1 to 100 is indicated on a drawing using the code 1:100. This can be interpreted as follows: 1 centimeter (0.01 meter) measured with a ruler on the plan

would need to be multiplied by 100 to give the actual size of 1 meter. So on a 1:200 scale plan, if you measured a wall length as 1 centimeter the actual length of the wall would be 2 meters.

Mechanical drawings at times are drawn to actual size (1 to 1) or even enlarged (2 to 1). These scales would be indicated on the drawing as 1:1 and 2:1.

The pictures below show measuring diagonally with a ruler from a wall corner on a site plan. If the measurement on the ruler is 78 millimeters (0.078 meters) and the draftsman is using a scale 1:200, then the actual size would be 15.6 meters (0.078m X 200).



How to draw a plan to scale

What do we mean by 'to scale'?

Plans drawn to a scale allow us to take measurements on the plans and convert them so we then know what sizes things will be in real terms. If your plan is not drawn to a scale you will need to mark all the relevant metric measurements on the plan.

Most commonly used scales are:

Scale 1:100 = 1cm on your plan denotes 1 meter in reality.

Scale 1:50 = 1cm on your plan denotes 50 cm in reality.

Before drawing a 'to scale' plan you need to consider both the size of the object you wish to draw and the size of the paper you wish to draw it on. If what you want to draw is very large, but your paper size is not, you will need a scale that turns the large in reality to the small on paper, such as 1:100.

As a Guide:

- **Site location plans** must be to the scale 1:1250 (we would recommend that you purchase rather than attempt to draw them yourself)
- **Site area plans** are usually drawn to the scale 1:500 (depending on the size of the whole plot)
- **Elevations** are usually drawn to scale 1:50 or 1:100
- **Internal/floor plans** are usually drawn to the scale 1:100 or 1:50

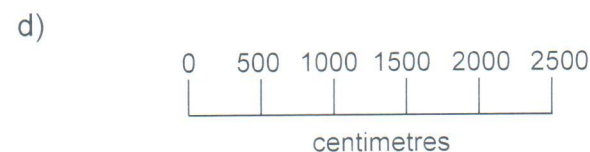
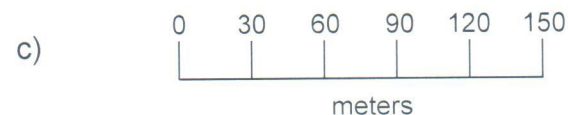
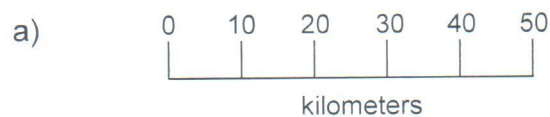
Once you have drawn your plans please mark clearly on the plan what scale it has been drawn to and what the paper size is, for example: "scale 1:100 at A4"

The above mentioned way of scaling is basically used for blueprints and other architectural plans. A more basic and simpler way of representing scales is through the use of the bar scale.

A **Bar Scale** is a way of indicating the scale used by means of a bar which shows the ratio of the measurement used.

Example 1

Write down the plan scales shown by the bar scales below.



Solutions:

- a) 1 centimetre on the map shows 10 kilometres on the ground, 2 centimetres on map shows 20 km on the ground... and so on
 so $10 \text{ km} = 10 \times 100\,000 \text{ cm}$
 $= 1\,000\,000 \text{ cm}$

Therefore, the scale used is 1:1 000 000.

- b) 1 centimetre on the map shows 20 meters on the ground, 2 centimetres shows 40 meters on the ground... and so on
 so $20 \text{ m} = 20 \times 100 \text{ cm}$
 $= 2\,000 \text{ cm}$

Therefore, the scale used is 1 : 2 000

- c) 1 centimetre on the map shows 30 meters on the ground, 2 centimetres shows 60 meters on the ground... and so on

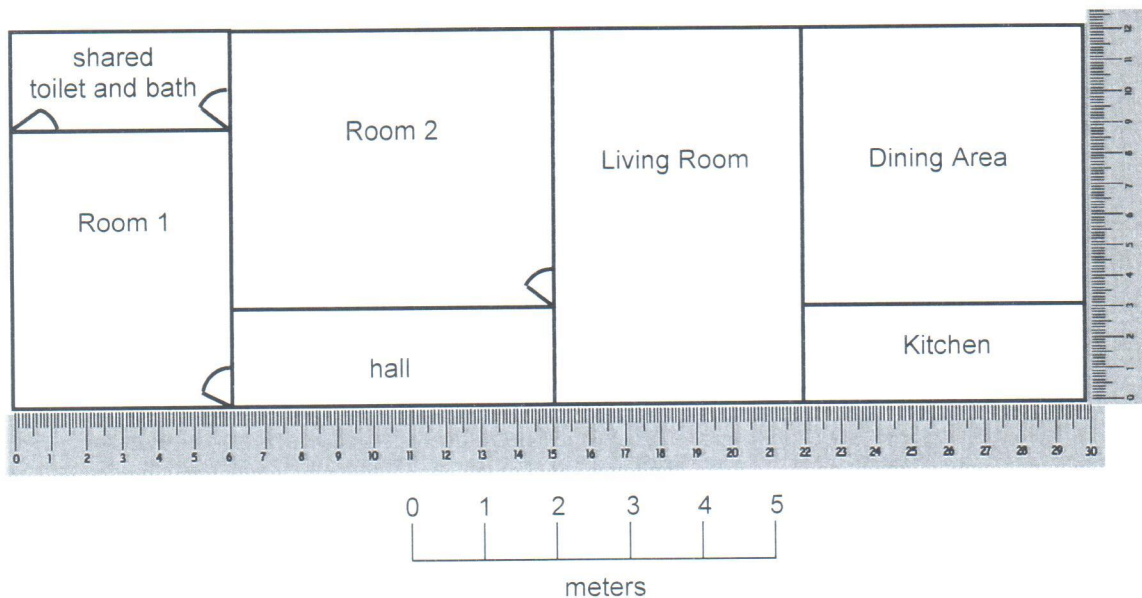
$$\begin{aligned}\text{so } 30 \text{ m} &= 30 \times 100 \text{ cm} \\ &= 3\,000 \text{ cm}\end{aligned}$$

Therefore the scale used is 1:3000.

- d) 1 cm on the map shows 500 cm on the ground, 2 cm on the map shows 1000 cm on the ground and so on... this time the same unit was used

Therefore the scale used is 1:500.

Example 2



- What scale was used?
- What are the dimensions of the lot in the plan?
- What is the actual area in m^2 of the lot on the ground?
- What is the actual distance in meter between Room 2 and the Dining Area?
- What is the area of the living room on the plan?
- Compute for the actual area of the living room in m^2 on the ground.

Solutions:

- Since $1 \text{ m} = 100 \text{ cm}$, we say $1 \text{ m} = 1 \times 100 \text{ cm} = 100 \text{ cm}$, therefore **the scale used is 1:100**. It means that every 1 cm in the plan corresponds with 100 cm (or 1m) on the ground.
- The length of the lot on the plan is 30 cm, while its width is 12 cm.

- c) To solve for the actual area, we first get the actual size of the lot in m^2 .

$$L = 30 \text{ cm (on the plan)} = 3000 \text{ cm} = 30 \text{ m (on the ground)}$$

$$W = 12 \text{ cm (on the plan)} = 1200 \text{ cm} = 12 \text{ m (on the ground)}$$

$$A = L \times W = 30 \text{ m} \times 12 \text{ m} = 360 \text{ m}^2$$

- d) Looking at the ruler, the distance between room 2 and the dining area marks in between 15 and 22.

To get the distance, we solve for their difference. $22 - 15 = 7$. It means that on the plan the distance between the two is 7 cm. We know that 1 cm on the plan represents 1 m on the ground. Therefore we say that the actual distance between room 2 and the dining area is 7 m.

- e) The areas of the living room on the plan are as follows.

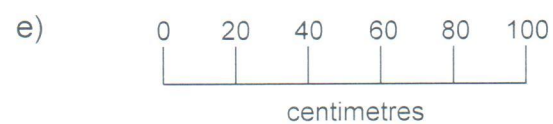
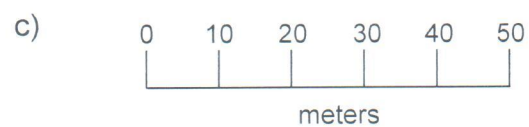
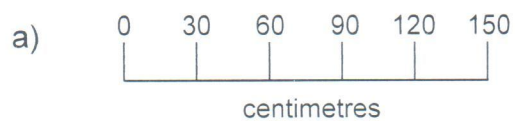
$$L = (22 \text{ cm} - 15 \text{ cm}) = \mathbf{7 \text{ cm}}; W = (12 \text{ cm} - 0 \text{ cm}) = \mathbf{12 \text{ cm}}.$$

- f) Since the length is 7cm and the width is 12 cm, we can say that on the ground their actual measures are 7m and 12m respectively. To solve for the area of the living room, we multiply the two, $7 \text{ m} \times 12 \text{ m} = \mathbf{84 \text{ m}^2}$.

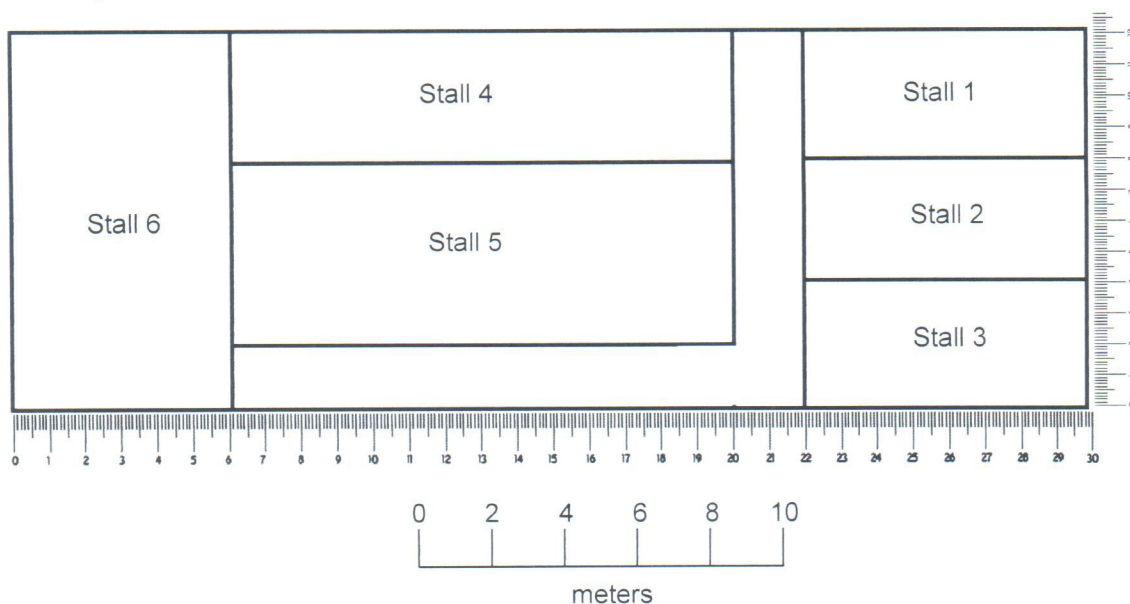
NOW DO PRACTICE EXERCISE 8

**Practice Exercise 8**

1. Write each bar scale in ratio form.



2. Using the plan below, answer the questions that follow:



a) What scale was used in the plan?

Answer: _____

b) What are the dimensions of Stalls 1, 2 and 3 on the plan?

Answer: _____

c) What is the actual floor area in m^2 of stall 1 on the ground?

Answer: _____

d) What is the area in cm^2 of Stall 6 on the plan?

Answer: _____

e) How far is stall 6 from stall 3 on the ground?

Answer: _____

f) What is the actual perimeter of stall 5 in meters?

Answer: _____

g) What is the actual area in m^2 of stall 4 on the ground?

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 9: Maps



You learnt something about plans in Lesson 8.



In this lesson you will:

- explain the usefulness of different types of maps
- solve problems based on maps.

Maps are graphic representations of places that use point, line, and area symbols, as well as color, to show how selected human and physical features are located, arranged, distributed, and related to one another.

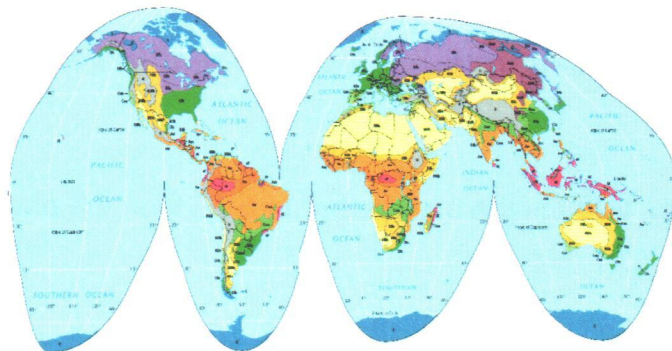


No single map can show everything, so the features portrayed on each map are selected to fit a particular purpose.

It is impossible to accurately depict the spherical Earth (globe) on a flat surface (map) without distorting shape, area, distance, or direction.



Globe



World Map

A map is a graphic representation of selected characteristics of a place, usually drawn to scale on a flat surface.

Please note that a person who makes maps is a **Cartographer**.



Map projection is the process of transferring information from a three-dimensional (spherical) surface to a two-dimensional (flat) surface. The pictures above are perfect examples. Every map has some distortion, either in shape, size (area), distance, or direction.

What are the Key Elements of a Map?

According to the National Geographic Map Making Guide, a good map should tell you what it is about (title), which direction north is (orientation), when the map was made or updated (date), who made the map (author), what the symbols mean (legend or key), how distances on the map relate to distances on the ground (scale), where to find selected places on the map (index), how to find places on the map (grid), and where the map's information comes from (sources or credits).



Remember! Not every map will identify all of the information stated above. The more information provided, the better you will be able to evaluate its content, credibility, and appropriateness for a given purpose or audience.

The acronym **DOGSTAILS** makes it easy to remember the important parts of a map:

D -	Date	-	When the map was made
O -	Orientation	-	Directions (north arrow)
G -	Grid	-	Locates places on the map
S -	Scale	-	Unit of measurement used to represent distance on the map
T -	Title	-	What, where, and when
A -	Author	-	Who made the map
I -	Index	-	Map address of places
L -	Legend	-	What the symbols used mean
S -	Sources	-	Basis for map information

Another acronym is **TODALSIGs** which makes it easier for us to remember the list of map elements in order of importance.

The acronym stands for

T -	Title	-	Text explaining what the map is supposed to portray: what is happening, where, and when.
O -	Orientation	-	In most cases, the direction "north" is assumed to be at the top of a map. A north arrow is a symbol indicating the direction in which north lies; a compass rose is a symbol indicating the cardinal directions (N, S, E, W) and sometimes directions (NE, NW, SE, SW).
D -	Date	-	Text identifying when the map was made and/or updated.

A -	Author	-	Text identifying the cartographer or organization responsible for making the map.
L -	Legend	-	Or known as Key is a guide identifying what the map's symbols and colors represent.
S -	Scale	-	The relationship between distance on a map and actual distance on the earth.
I -	Index	-	A listing of the places on the map and where to find them using <i>grid</i> coordinates either <i>latitude-longitude</i> (77°53'W, 38°02'N) or letter-number (B4).
G -	Grid	-	Intersecting lines (called a "coordinate system") drawn on a map to pinpoint location.

Any of these two acronyms will help you remember easier and faster the important parts of a map.

Other important features that could be seen on a map are the following:

Neat line: A box drawn around the map to give a "neat," or tidy, appearance to the product.

Insets: Additional, smaller maps set within the larger main map.

Graphs: Pictorial representations of numeric data, often used as an additional method of displaying the data represented on the map. Line graphs, bar graphs, and pie charts are common types.

Timelines: Text and graphics arranged sequentially along a line to give information about when events or phenomena occurred. Timelines are sometimes used on maps to give a better idea of how time relates to the data or theme represented.

Explanatory Text: Text that is not part of the map's title, legend, or credits, used to give the map reader a deeper understanding of the map, its data, and/or its theme.

Credits: Text that may include sources, author, photographers, individuals, or organizations involved in creating the map or its contents.

Types of Maps

There are two main types of maps, based on their design purpose. Reference (or general purpose) maps depict selected details of the physical and human-made environment as accurately as possible. Thematic (or special purpose) maps depict the general spatial pattern of selected features or data.

1. Reference Maps

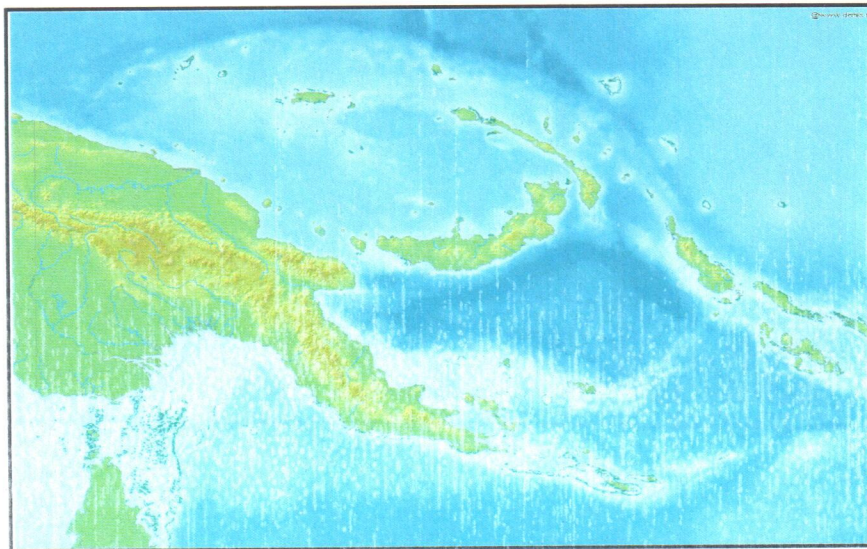
Reference or general purpose maps show selected features of the physical and human-made environment. Reference maps use symbols to locate and identify important landmarks and geographic features.

Examples include road maps, classroom wall maps, political maps, physical maps, and topographic maps.

- 1.1 Political Maps** are reference maps that show names and boundaries of political geographic units (states, countries, etc.) and only very important physical or human features (e.g., rivers, highways, etc.)



- 1.2 Physical Maps** use shaded or painted relief to illustrate the major landforms (natural features) of a region, country, or world. Mountain ranges and rivers usually are prominent features on such maps; other features would include deserts, glaciers, lakes, and valleys. The colors on physical maps often include brown or green for land and blue for water.



- 1.3 Topographic Maps** are general reference maps showing coastlines, cities, and rivers that use contour lines to show elevation differences. Such maps are helpful to hikers because they can show elevation changes along a trail. Government agencies like the U.S. Geological Survey (USGS) produce these maps that are often based on satellite data or aerial photography.



- 1.4 Satellite Maps** are produced from data recorded by satellite sensors and transmitted to Earth where computers process the data into images. Satellites can provide map images of formerly inaccessible parts of our world and of distant worlds—like Mars or the Moon.



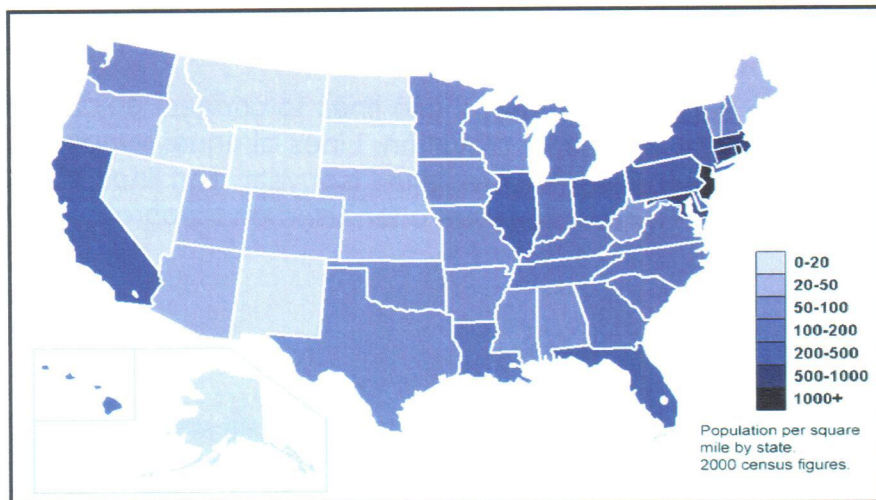
2. Thematic (Special Purpose) Maps

Thematic or special purpose maps display distributions or patterns over Earth's surface. Thematic maps emphasize some particular feature or set of data such as rainfall or locations of crops, using colour, shading, or symbols to represent differences.

Examples include census maps which focus on population distribution as well as data on such items as age, ethnicity, and income; these maps help governments provide services to its citizens and plan for the future.

Thematic maps can portray data using shaded areas (choropleth), dots (dot-density), symbols of different sizes (graduated symbol), lines of equal measurement (isoline), or proportional size of area (cartogram).

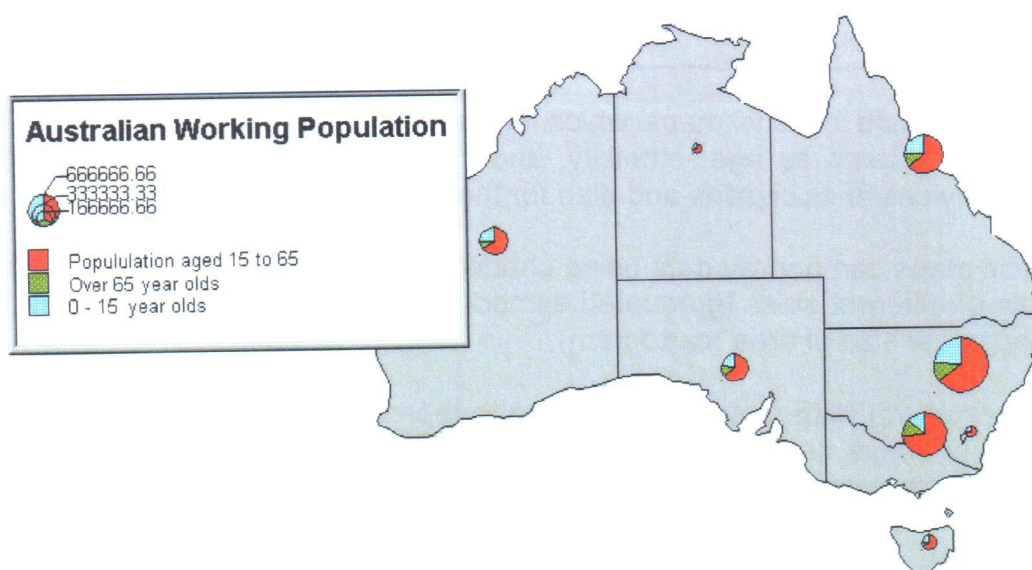
2.1 Choropleth Maps use color shading to represent different quantities or values. Darker colors usually represent greater quantities or values, while lighter colors usually represent smaller quantities or values.



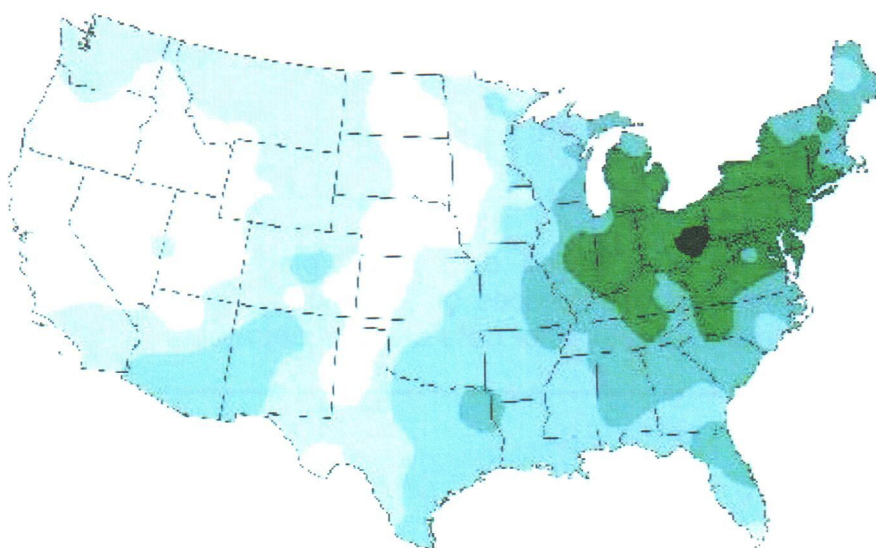
2.2 Dot-density (also called dot) Maps place individual points on a map to correspond with occurrences of a particular feature or data. Clusters of dots show where the features or data are concentrated.



- 2.3 Graduated symbol maps** use symbols of different sizes placed within an area to show the value or quantity associated with it. The symbol is often a circle, but it may be one that relates to the map's theme (such as figures of people to represent population data).

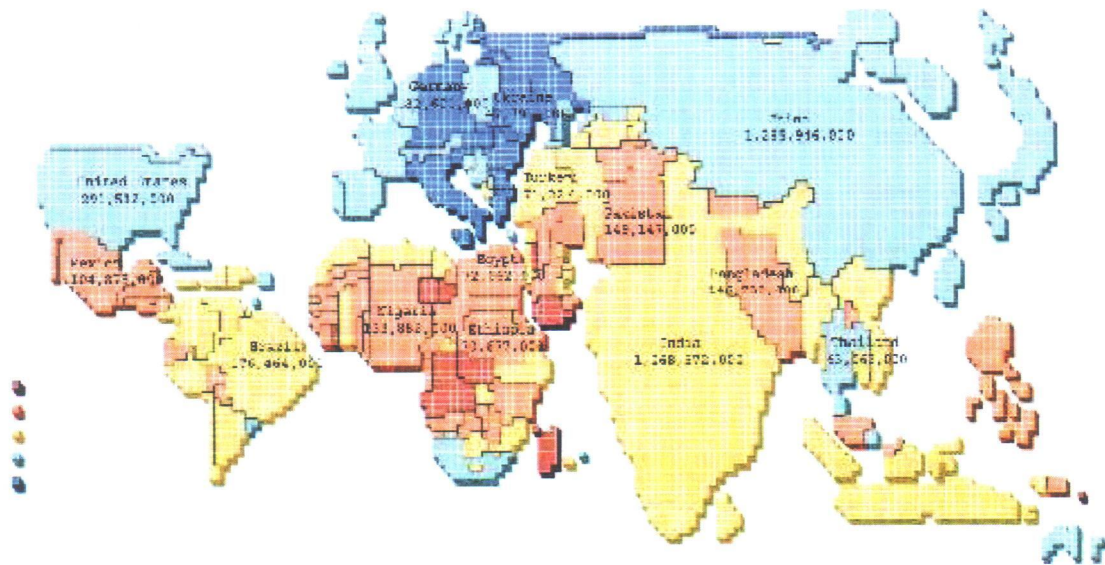


- 2.4 Isoline** (also called **isopleth**) **maps** use lines to connect points of equal value, such as temperature, rainfall, or elevation. Lines of equal temperature are called isotherms, lines of equal rainfall are called isohyets, and lines of equal elevation are called contours. Contour lines are often used on topographic maps.



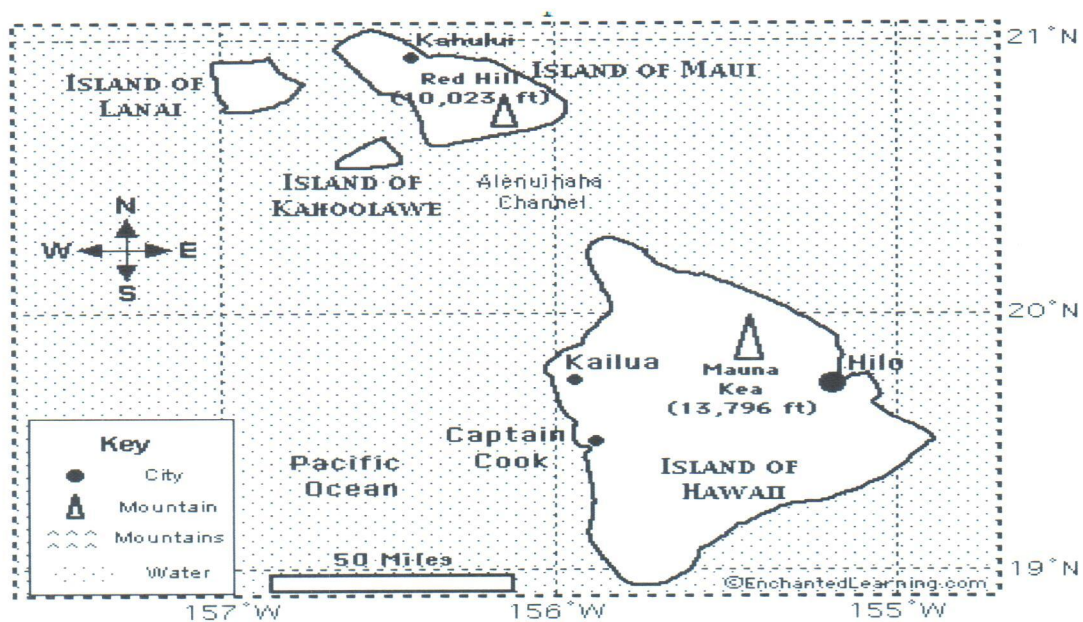
- 2.5 Cartograms** distort the size and shape of map areas to show statistical data. For example, a cartogram of world population shows land area as proportional to population size—although the United States and China have similar land areas (the U.S. is slightly larger), a population cartogram would show China as much larger than the U.S. because its population is much larger.

(See the map on the next page)



Example

Use the map below, to answer the following questions.



- What is the name of the largest island on the map?
- Name the ocean on the map.
- Which island is farthest west?
- Which island is farthest south?
- What mountain is located at latitude 19.75°N, longitude 155.5°W?
- What direction would you travel to go from Hilo to Mauna Kea?

Solution:

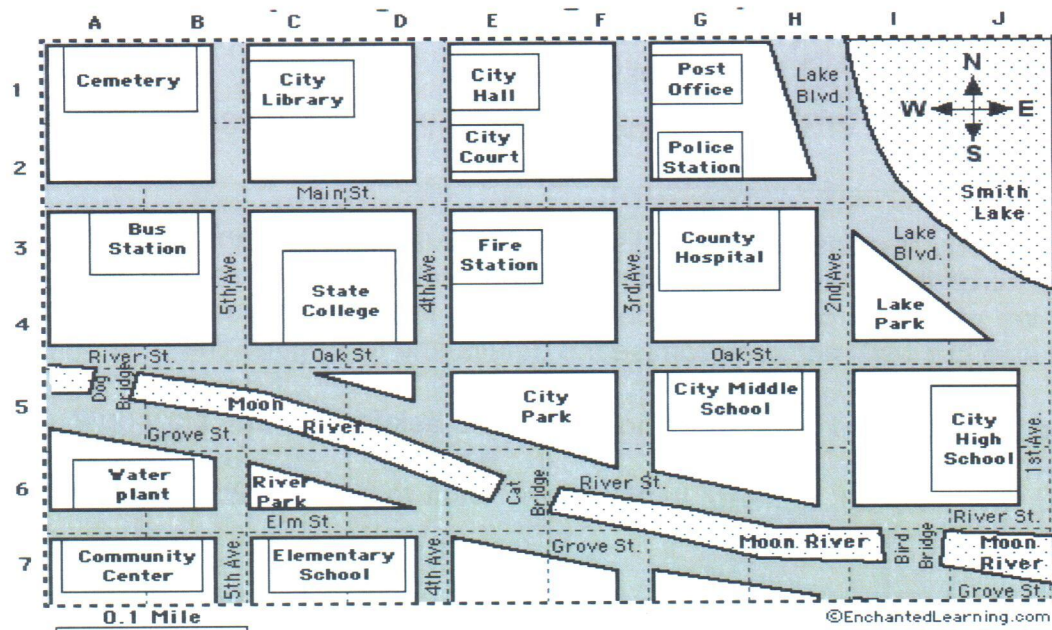
- a) Island of Hawaii
- b) Pacific Ocean
- c) Island of Lanai
- d) Island of Hawaii
- e) Mauna Kea
- f) North West

NOW DO PRACTICE EXERCISE 9



Practice Exercise 9

Refer to the map below to answer the following questions.



- Which is farthest east, the Elementary or Middle School?
Answer: _____
- Which is farther south, Lake Park or City Park?
Answer: _____
- How far in miles is it from the Bus Station to the Hospital?
Answer: _____
- Which school's address is 250 Oak Street?
Answer: _____
- What city building is located at E3?
Answer: _____
- When you look northeast, you see River Park. When you look north, you see Water Plant. When you look east, you see the Elementary School. Where are you?
Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 10: Enlargements and Similar Shapes



You learnt the different types of maps and their uses in the last lesson.



In this lesson, you will:

- define Enlargement and Similar Shapes.

In the previous lessons, we have discussed about models, plans and maps. If you noticed, all of them involved re-sizing the original objects to smaller one. In this lesson, we will focus on re-sizing objects bigger than the original or enlarged to its original size but it will not compromise the details and its original characteristics.

Have you experienced going to a photo shop and ask for a photo “blow up”?

When u say “blow up” a photo, it means you want to have another print bigger than the original.

Observe the photos below. On top is the original photo of a man. Below are three enlarged photos of the original.



original photo



A



B



C

What do you notice about the three enlarged photos?

Which one of them looks exactly the original?

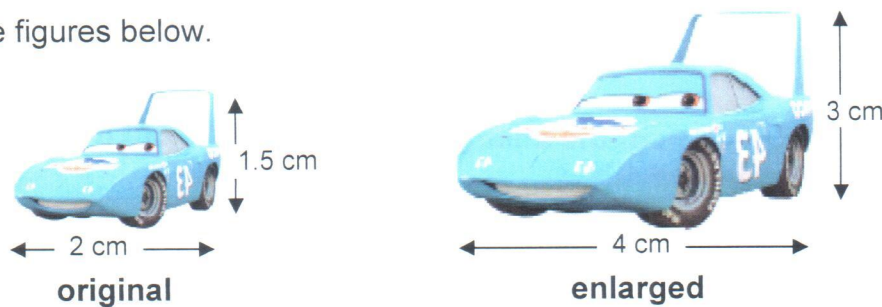
Very keen observer, if you noticed that photo B looks exactly the same yet bigger than the original one.

What happened to photos A and B?

Photo A had too much adjustment in terms of its width. The guy in the picture looked chubbier. On the other hand, photo B had too much adjustment in terms of its height. The guy in the picture looked skinny or thin.

When an object drawn is enlarged, a scale must be used to avoid distortion of the original figure and to ensure that an exact image will be shown.

Observe the figures below.



The length of the original drawing is 2 cm while in the enlarged drawing measures 4 cm. The height of the original is 1.5 cm and the enlarged height is 3 cm.

What do you notice about the length and height of the enlarged drawing in comparison with the original's dimensions (length and width)?

Aside from the fact that dimensions of the enlarged figure are always greater than the dimensions of the original, notice in particular that, both dimensions were doubled in measures or the dimensions of the enlarged drawing are twice the originals.

Do you agree?

Great! Then we say that the drawing is enlarged by scale factor 2. Remember our lesson about scale factor in Unit 5? Yes, this is one of its many applications.

For all enlargements, always use a scale factor greater than 1.

If you use a scale factor of 1, it is not considered as an enlargement because you just duplicated the original. While if a scale factor less than 1 is used, the figure will not be enlarged but it will be reduced instead.

In order for us to get the measures of the enlarged object, we multiply the measure of the original object with the set scale factor. In symbol we say:

Measure of the enlarged object denoted as : $\text{Length}_{(\text{enlarged})}$

$$\text{Length}_{(\text{enlarged})} = \text{Length}_{(\text{original})} \times \text{Scale Factor}$$

Example 1

A cylinder having a diameter of 5 cm and height of 12 cm is to be enlarged thrice its size. What are the dimensions of the enlarged cylinder?

Solution:

First, we identify the given information in the problem:

Diameter of the cylinder = 5 cm; height of the cylinder = 12 cm

Enlargement Scale factor = 3 (since it was mentioned that it must be **thrice** the original)

Knowing that a scale factor of 3 will be used, we multiply each dimension of the cylinder by 3:

$$\text{Diameter} = 5 \text{ cm} \times 3 = 15 \text{ cm}$$

$$\text{Height} = 12 \text{ cm} \times 3 = 36 \text{ cm}$$

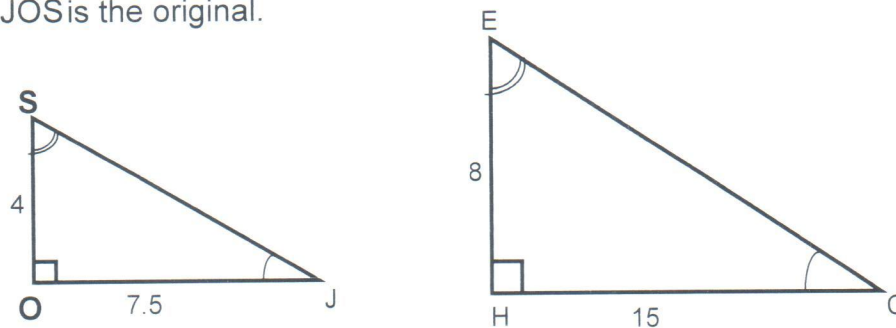
Therefore, the dimensions of the enlarged cylinder are 15 cm (diameter) and 36 cm (height) respectively.

Enlargements and Similarity

In Unit 5, we discussed the meaning of similarity when we learnt about similar triangles. We even emphasized that the word similar is sometimes synonymously used as the word “same”. Others confusingly use it to replace the word congruent. These words may be synonymous in a way in written English but in Mathematics they really mean different.

Shapes are similar if the ratio of their corresponding sides (dimensions) is proportional.

Let us observe the following, given two right triangles where $\triangle CHE$ is the enlarged triangle and $\triangle JOS$ is the original.



A proportion can be formed through the pairs of corresponding sides

$$\frac{\overline{EH}}{\overline{SO}} = \frac{\overline{EC}}{\overline{SJ}} = \frac{\overline{HC}}{\overline{OJ}} \quad \text{or can also be written as} \quad \frac{\overline{SO}}{\overline{EH}} = \frac{\overline{SJ}}{\overline{EC}} = \frac{\overline{OJ}}{\overline{HC}}$$

The first set will give you a ratio greater than 1 because we used the values of the enlarged triangle first. In that case, the ratio derived is also the enlargement scale factor.

On the other hand, if the second set of proportions will be used, it will show equal ratios but the ratio formed is definitely a fraction. This fraction suggests the scale factor for reduced objects.

Using the following $\frac{\overline{EH}}{\overline{SO}} = \frac{\overline{EC}}{\overline{SJ}} = \frac{\overline{HC}}{\overline{OJ}}$, we say that $\overline{EH}$ from the bigger triangle corresponds with $\overline{SO}$ from the original triangle, $\overline{EC}$ corresponds with $\overline{SJ}$ and $\overline{HC}$ corresponds with $\overline{OJ}$.

Since both triangles are right, with the similarity postulates you learned in Unit 5, we can test their similarity by forming proportions using the measures of their legs.

So, we say:

$$\frac{\overline{EH}}{\overline{SO}} = \frac{\overline{HC}}{\overline{OJ}}$$

Substitute the values:

$$\frac{8}{4} = \frac{15}{7.5}$$

Simplify:

$$2 = 2$$

Since $2 = 2$, then it proves that the ratio of their sides forms a proportion. We can also say that the scale factor is 2.

Example 2

The scale factor for a model unit is 5. If the model measures 9.5 cm, what is the measure of the enlarged model?

Solution:

Earlier, we discussed that in order for us to get the measures of the enlarged object, we multiply the measure of the original object with the set scale factor. In symbol, we say:

Given: $\text{Length}_{(\text{original})} = 9.5 \text{ cm}$; Scale Factor = 5

Use the formula: $\text{Length}_{(\text{enlarged})} = \text{Length}_{(\text{original})} \times \text{Scale Factor}$

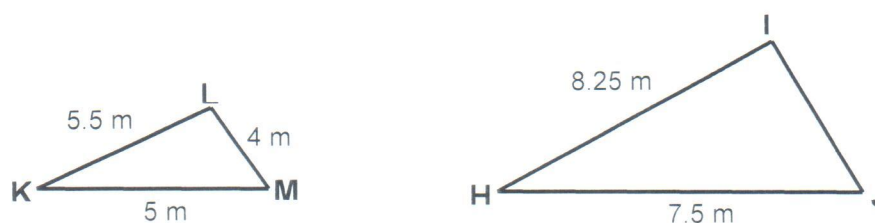
Substitute: $\text{Length}_{(\text{enlarged})} = 9.5 \text{ cm} \times 5$

Simplify: $\text{Length}_{(\text{enlarged})} = 47.5 \text{ cm}$

Therefore, the measure of the enlarged model is 47.5 cm.

Example 3

Triangle KLM is enlarged forming triangle HIJ as shown in the figure below:



- What is the enlargement scale factor?
- What is the measure of side IJ?

Solution:

a) Using the proportion $\frac{HI}{KL} = \frac{HJ}{KM}$ we form $\frac{7.5 \text{ m}}{5 \text{ m}} = \frac{8.25 \text{ m}}{5.5 \text{ m}}$

We simplify to prove if the ratios form a proportion and to solve for the scale factor as well.

$$\frac{7.5 \text{ m}}{5 \text{ m}} = \frac{8.25 \text{ m}}{5.5 \text{ m}} \quad \begin{array}{l} \text{divide } 7.5 \div 5 = 8.25 \div 5.5 \\ \text{simplify } 1.5 = 1.5 \end{array}$$

Therefore, the scale factor is 1.5.

- b) To solve for the measure of IJ, we use the given information we derived in (a) that is scale factor is 1.5. Looking at the figure, LM corresponds to IJ, therefore we enlarge the value of LM to make it equal to IJ.

Given: Length_(LM) = 4 cm ; Scale Factor = 1.5

Use the formula: Length_(IJ) = Length_(LM) x Scale Factor

Substitute : Length_(IJ) = 4 cm x 1.5

Simplify: Length_(IJ) = 6 cm

Therefore, the measure of side IJ is 6 cm.

NOW DO PRACTICE EXERCISE 10

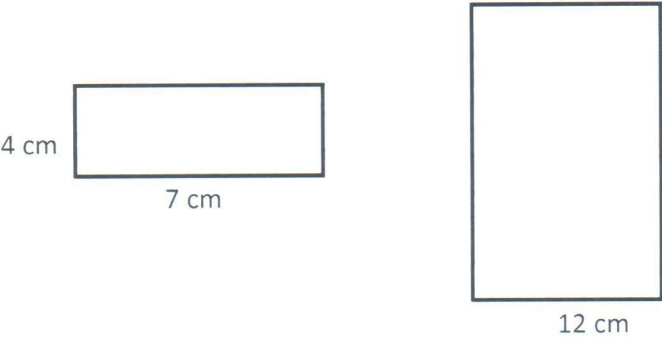


Practice Exercise 10

1. Rectangles were enlarged to the following dimensions. Complete the missing information on the table below: (Express answer to two decimal places).

Original		Enlarged		Scale Factor
length	width	length	Width	
15 cm	4 cm			4
		16.2	8.8	0.25
4		65	10	
12	8		14	
8.15			18.27	3.55

2. Use the given information below to answer the questions that follows:



- a) What is the scale factor used?
Answer: _____
- b) What is the length of the enlarged rectangle?
Answer: _____
- c) What is the area of the enlarged rectangle?
Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 11: Area of Similar Shapes



You learnt about similar shapes and calculating areas of plane figures in the previous uUnits in this course.



In this lesson, you will:

- calculate area of similar shapes.

In the previous lessons, we discussed that similar shapes are proportional with each other and that a similar shape can be produced by using a scale factor. Scale factor was discussed in Lesson 7 of this Unit book.

A rectangle with sides measuring 1 cm and 3 cm is drawn below.



We know that its area is computed by multiplying the length and width ($A = 1 \text{ cm} \times 3 \text{ cm}$), thus the area of the rectangle is 3 cm^2 .

If a similar rectangle is drawn using the scale factor 2, we will have the following figure.



So we can now say that the area of the enlarged rectangle is 12 cm^2 ($A = 2 \text{ cm} \times 6 \text{ cm}$).

Now let us organize our observations and try to analyse the table below which shows the dimensions, scale factors and areas of rectangles similar to the original rectangle drawn earlier.

Width	Length	Scale Factor	Area
1 cm	3 cm	Original Figure	3 cm^2
2 cm	6 cm	2	12 cm^2
5 cm	15 cm	5	75 cm^2
15 cm	45 cm	15	675 cm^2

Finding the area of the rectangles similar to given is easy because you just have to multiply the width and the length. But as we analyse the given, we will try to come up with a generalized rule which will help us solve further areas of other similar shapes.

Looking at the scale factor and areas computed, do you see any relationship?
Using a scale factor of 2, how do we get 12?

Can we think this way?

Scale Factor	Analysis	Area
Original Figure		3 cm ²
2	2 ² (3cm)	12 cm ²
5	5 ² (3cm)	75 cm ²
15	15 ² (3cm)	675 cm ²

Did you see the pattern?

The following conclusion can be drawn from the table above.

The Area of Similar Shapes is computed by multiplying the square of the scale factor with the area of the original shape.

$$\text{Area of Similar Shapes} = (\text{scale factor})^2 \times (\text{area of original shape})$$

In which way does this formula help?

This can be used in computing for areas of enlarged or reduced models whose dimensions (length, width, height) are not indicated.

Example 1

A triangle has an area of 45 cm². Find the area of a similar triangle enlarge using a scale factor of 7.

Solution:

This problem requires the use of the formula derived.

$$\text{Area of Similar Shapes} = (\text{scale factor})^2 (\text{area of original shape})$$

$$\text{Area of Similar Shapes} = (7)^2 (45\text{cm}^2) = 49 (45\text{cm}^2) = 2205 \text{ cm}^2$$

Example 2

A square has a side length of 5 m. Find the area of the enlarged model scaled at 12.5.

Solution:

We have to solve for the area of the original figure before we can use the formula. Since the area of the square is s², we say that Area of Square is (5m)² = 25 m².

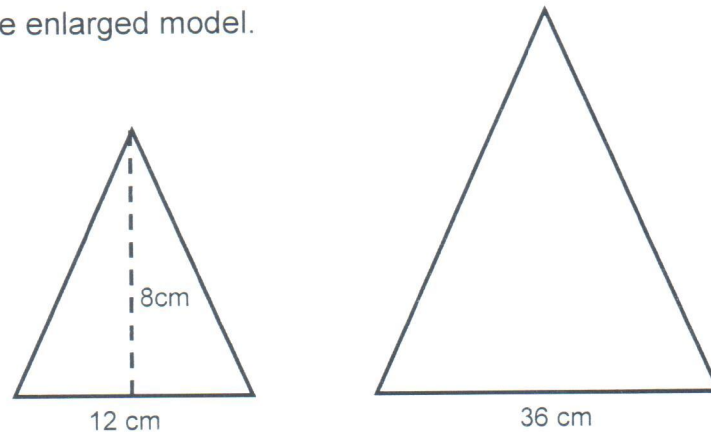
Using the formula:

$$\text{Area of Similar Shapes} = (\text{scale factor})^2 (\text{area of original shape})$$

$$\text{Area of Similar Shapes} = (12.5)^2 (25\text{m}^2) = 156.25 (25\text{m}^2) = 3906.25 \text{ m}^2$$

Example 3

Find the area of the enlarged model.



Solution:

In this case, we need to solve for the area of the original figure and the scale factor used before we can apply the formula.

$$\text{Area of the triangle} = \frac{1}{2}bh = \frac{1}{2}(12 \text{ cm})(8\text{cm}) = 48 \text{ cm}^2.$$

To solve for the scale factor, (recall Lesson 10) or simply used the following ratios of side length:

Enlarge shapes: $\frac{\text{enlarged shape}}{\text{original shape}}$

Reduced shapes: $\frac{\text{reduced shape}}{\text{original shape}}$

$$\text{Since we used an enlarged model, we have: } \frac{36\text{cm}}{12\text{cm}} = 3$$

Now we can use the formula:

$$\text{Area of Similar Shapes} = (\text{scale factor})^2 (\text{area of original shape})$$

$$\text{Area of Similar Shapes} = (3)^2 (48\text{cm}^2) = 9 (48 \text{ cm}^2) = 432 \text{ cm}^2$$

Therefore, the area of the enlarged triangle is 432 cm^2 .

NOW DO PRACTICE EXERCISE 11

**Practice Exercise 11**

1. The area of a pentagon is 12.25 cm^2 . Find the areas of the following similar pentagons enlarged / reduced using the following scale factors:

a) 12

Answer: _____

b) 10

Answer: _____

c) $\frac{1}{3}$

Answer: _____

d) 0.75

Answer: _____

2. A rectangle has the following dimensions: $l = 15 \text{ cm}$; $w = 42 \text{ cm}$.

a) Find the area of a similar rectangle enlarged using a scale factor of 6.

b) Find the area of a similar rectangle whose length is 3 cm.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.
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TOPIC 2: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- A scale model is a representation or copy of an object which shows accurate relationship between all important aspects of the object.
- Similar objects are two or more objects whose corresponding parts (sides) form a proportion.
- A plan is a complete illustration of the important parts of the original object or structure which includes the measurement, the shape, the design and other specific details.
- Plans drawn to a scale allow us to take measurements on the plans and convert them so we then know what sizes things will be in real terms.
- A Bar Scale is a way of indicating the scale used by means of a bar which shows the ratio of the measurement used.
- Maps are graphic representations of places that use point, line, and area symbols, as well as colour, to show how selected human and physical features are located, arranged, distributed, and related to one another.
- For all enlargements, always use a scale factor greater than 1.
- Shapes are similar if the ratio of their corresponding sides (dimensions) is proportional.

REVISE LESSON 1 TO 6. THEN DO TOPIC TEST 2 IN ASSIGNMENT BOOK 6.

ANSWERS TO PRACTICE EXERCISES 7 – 11

Practice Exercise 7

- a) dimensions of big photo: 120 inches by 48 inches.
 - a) 1 : 5 b) 2.4 ft c) 8 ft
 - a) 96.25 m b) 7.6 cm c) 75 m by 110 m d) 8250 m²
 - a) 270 m b) 117 m
-

Practice Exercise 8

- a) 1: 30 b) 1: 500 c) 1:1000
 d) 1: 200 000 e) 1: 20 f) 1 : 400
 - a) 1 : 200 b) 4 m by 2 m c) 8 m²
 d) 80 000 cm² e) 30 m f) 18 m g) 14 m²
-

Practice Exercise 9

- a) Elementary School
 b) City Park
 c) 0.4 miles
 d) City Middle School
 e) Fire Station
 f) Community Centre
-

Practice Exercise 10

- | Original | | Enlarged | | Scale Factor |
|-------------|-------------|--------------|-----------|--------------|
| length | width | Length | width | |
| 15 cm | 4 cm | 60 | 16 | 4 |
| 64.8 | 35.2 | 16.2 | 8.8 | 0.25 |
| 4 | 0.63 | 65 | 10 | 16.3 |
| 12 | 8 | 21 | 14 | 1.75 |
| 8.15 | 5.15 | 28.93 | 18.27 | 3.55 |

- a) 3 b) 21 c) 252 cm²

Practice Exercise 11

1. a) 147 cm^2
b) 122.5 cm^2
c) 4.08 cm^2
d) 9.19 cm^2
 2. a) 3780 cm^2
b) 3.21 cm^2
-

END OF TOPIC 2

TOPIC 3

AREA

Lesson 12:	Area
Lesson 13:	Surface Area of Prisms, Cubes and Cuboids
Lesson 14:	Surface Area of Triangular and Trapezium Prisms
Lesson 15:	Surface Area of Pyramids
Lesson 16:	Surface Area of Cylinders
Lesson 17:	Surface Area and Problem Solving

TOPIC 3: AREA

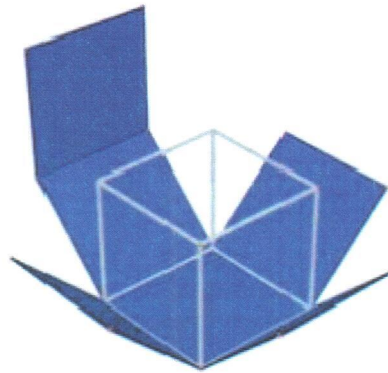
Introduction



In your lower grades Mathematics, you have learnt how to solve for areas of different two dimensional figures such as polygons and the circle as well as irregular-shaped figures.

In this topic, you will revise your knowledge in solving for the area and use it to derive formulas in solving for the surface areas of three-dimensional figures.

We will also explore solid figures by examining their nets.



The skills you will develop in this topic will not only enhance your analytical thinking but will help you develop more accuracy in deriving and using related formulas.

In addition, you will also solve real – life problems involving the surface areas of three-dimensional or solid figures that we commonly see around us and use in our daily living.

Lesson 12: Area



You have learned something about area in your earlier Mathematics.



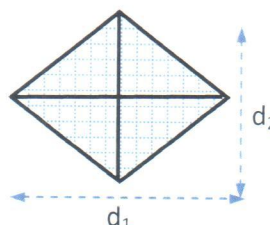
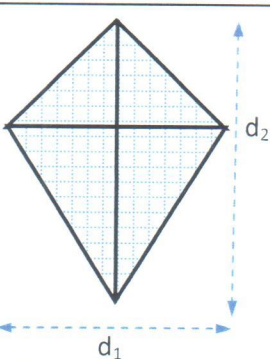
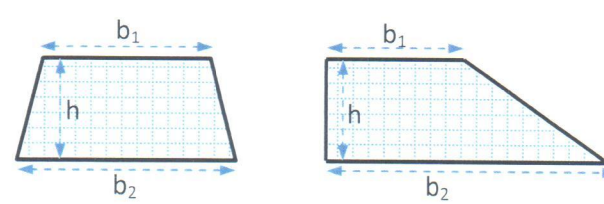
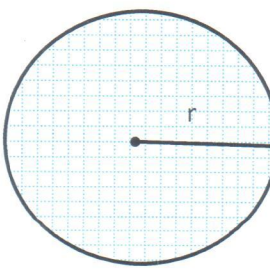
In this lesson, you will:

- calculate the area for composite shapes using correct units.

In your previous Mathematics, you learnt about the formulas used in solving for the area of plane figures. In this lesson we will review them first because they will be used in solving for the area of irregular shapes.

Recall the following formulas:

Plane Figure	Formula	Sample Figures
Triangle	$A = \frac{(b)(h)}{2}$ where: b= base h= height	
Rectangle	$A = (b)(h)$ where: b= base h= height	
Parallelogram	$A = (b)(h)$ where: b= base h= height	
Square	$A = s^2$ where: s = length of one side	

Rhombus	$A = \frac{1}{2}(d_1)(d_2)$ where: d_1 = first diagonal d_2 = second diagonal	
Kite	$A = \frac{1}{2}(d_1)(d_2)$ where: d_1 = first diagonal d_2 = second diagonal	
Trapezium (Trapezoid)	$A = \frac{(b_1 + b_2)(h)}{2}$ where: b_1 = upper base b_2 = lower base h = height	
Circle	$A = \pi r^2$ where: r = radius	

Composite Shapes

The following are examples of composite shapes. Observe them carefully and try to compare them with the plane figures reviewed above.

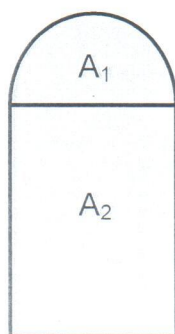


Figure 1

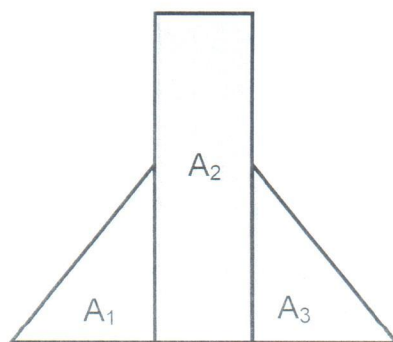


Figure 2

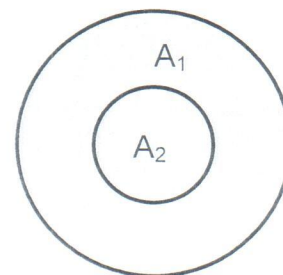


Figure 3

Did you notice that the composite shapes are made up of different plane figures?

Composite Shapes are plane figures which are irregular in shape. Basically the composite shape is composed of a combination of two or more polygons or a circle.

In Figure 1, to get the Area (shaded part) of the composite shape we add the areas of the semi-circle and the rectangle. We represent the area using the formula:

$$A = A_1 + A_2$$

In Figure 2, how many polygons are there?

There are two triangles and a rectangle that comprise the shape. Therefore, we add the area of the three polygons and represent the area as

$$A = A_1 + A_2 + A_3$$

How about in Figure 3?

Did you notice two overlapping circles?
How to get the area of the shaded portion?
Are we going to add the area of the two?

You are thinking critically if you say that we cannot add the area of the two because the inner or smaller circle is not shaded. What are you going to do then?

Yes, you are right if you say that we get the difference of the areas of the two circles. We can now represent the area as

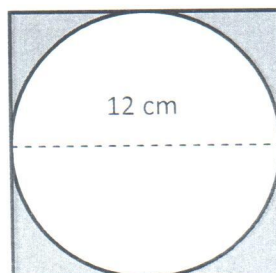
$$A = A_1 - A_2.$$

You see, getting the area of composite figures is very easy. All you have to do is to be very keen in identifying the polygons that make the composite shape and remember the formulas in solving for the area of these basic polygons.

Study the following illustrative examples.

Example 1

Find the area of the shaded part of the square. (Use $\pi = 3.14$)



Solution:

There are two plane figures involved. The first one is a square and inside it is a circle. Since the circle is inside the square and we see that the circle is tangent to each side of the square, we can now consider that the diameter of the circle is equal to the length of each side of the square. This information helps to solve the problem.

First, we get the area of the square.

Recall: $A = s^2$, where s is the length of each side of the square.

In the figure, $s = 12$ cm

We substitute 12 cm in the formula: $A = (12 \text{ cm})^2$

or simply say

$$\begin{aligned} A &= (12\text{cm})(12\text{cm}) \\ &= 144\text{cm}^2. \end{aligned}$$

Next, we get the area of the circle. Recall: $A = \pi r^2$, where r is the radius.

Since the given 12 cm is the diameter of the circle, we get half of it to represent the length of the radius. So, we say, $r = 6$ cm.

We substitute 6 cm in the formula: $A = \pi(6\text{cm})^2$

$$\begin{aligned} &= (3.14)(36\text{cm}^2) \\ &= 113.04 \text{ cm}^2. \end{aligned}$$

Finally, we get the difference of the areas of the two.

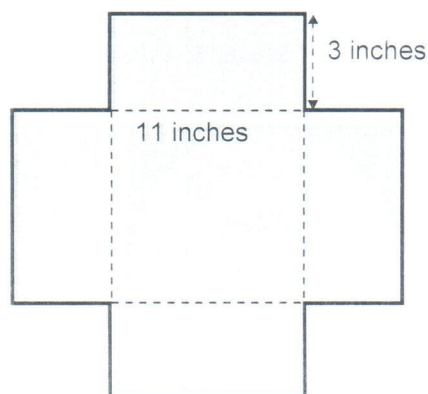
Area of the shaded part = Area of the square – Area of the circle

$$\begin{aligned} A &= 144\text{cm}^2 - 113.04\text{cm}^2 \\ &= 30.96 \text{ cm}^2 \end{aligned}$$

Therefore, the area of the shaded portion of the square is 30.96 cm^2 .

Example 2

A square base box is to be made using a piece of cardboard. If the square base has a side length of 11 inches and the box must have a height of 3 inches, what is the area of the cardboard to be used?



Solution:

Since the base of the box is a square, we use the formula s^2
The faces of the box are rectangles which are equal in size, so we use $4(L \times W)$

Solving for the area of the cardboard we use:

$$A = s^2 + 4(L \times W)$$

Substitute in the formula $s = 11$ inches, $l = 11$ inches, $w = 3$ inches

$$A = (11 \text{ inches})^2 + 4(11 \text{ inches} \times 3 \text{ inches})$$

Simplify:

$$A = 121 \text{ inches}^2 + 4(33 \text{ inches}^2)$$

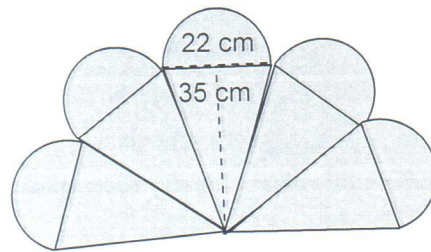
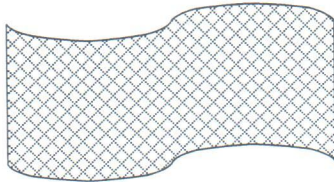
$$= 121 \text{ inches}^2 + 132 \text{ inches}^2$$

$$= 253 \text{ inches}^2$$

Therefore, the area of the cardboard to be used is 253 inches².

Example 3

A student would like to know if a gift wrapping paper whose area is 3000 cm² is enough to cover a fan made up of cardboard with the following measurement: (Use $\pi = 3.14$)



Solution:

Observe that the fan is made up of five (5) cone shape with congruent measures. Each cone shape is made up of a semi-circle and a triangle. For the semi-circle we divide the formula for the area of the circle by 2.

We will now use $\frac{\pi r^2}{2}$. For the triangle we will use $\frac{bh}{2}$.

We can now say that area of one cone is $\frac{\pi r^2}{2} + \frac{bh}{2}$

Since there are 5 cones, we multiply this by 5.

$$A = 5 \left(\frac{\pi r^2}{2} + \frac{bh}{2} \right)$$

We use $r = 11$ cm (half the diameter)
 $b = 22$ cm; $h = 35$ cm

Substitute in the formula:

$$A = 5 \left(\frac{(3.14)(11\text{cm})^2}{2} + \frac{(22\text{cm})(35\text{cm})}{2} \right)$$

Simplify:

$$A = 5 \left(\frac{379.94\text{cm}^2}{2} + \frac{770\text{cm}^2}{2} \right)$$

$$\begin{aligned} A &= 5 (189.97 \text{ cm}^2 + 385 \text{ cm}^2) \\ &= 5 (574.97 \text{ cm}^2) \\ &= \mathbf{2874.85 \text{ cm}^2} \end{aligned}$$

Since the area of the fan is 2874.85, the special paper is enough to cover the fan.

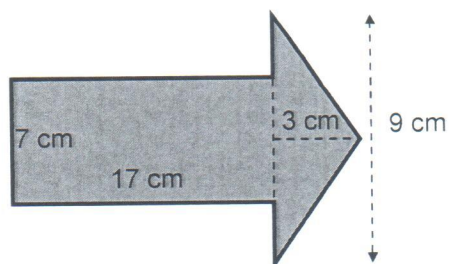
NOW DO PRACTICE EXERCISE 12



Practice Exercise 12

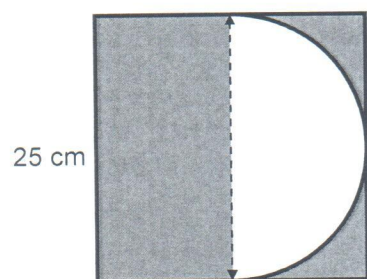
Find the area of the shaded part in the following figures. (Use $\pi=3.14$)

1.



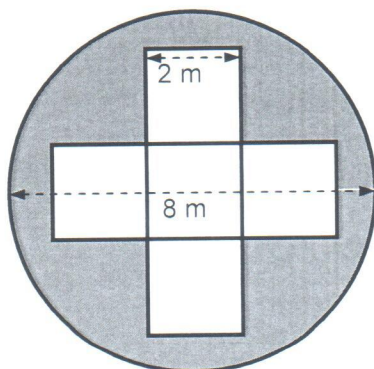
Answer: _____

2.



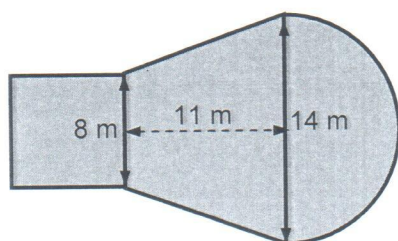
Answer: _____

3.



Answer: _____

4.



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 13: Surface Area of Prism, Cubes and Cuboids



You learnt to workout areas of composite shapes in Lesson 12..

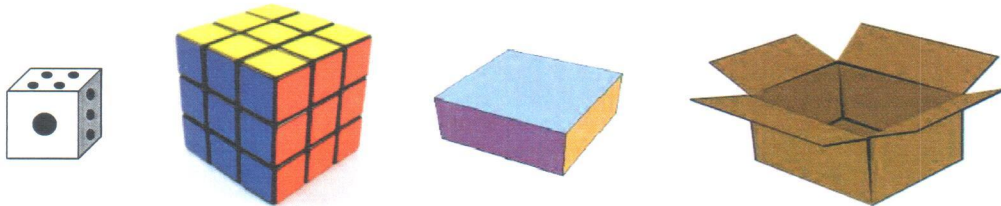


In this lesson, you will:

- identify the surface area of open and closed cubes and cuboids
- calculate the surface area of open and closed cubes and cuboids.

In your previous Mathematics courses, you learnt what solid figures are particularly prisms. In this lesson, we will have a closer look at these solids specifically cubes and cuboids.

Are you familiar with the following objects?

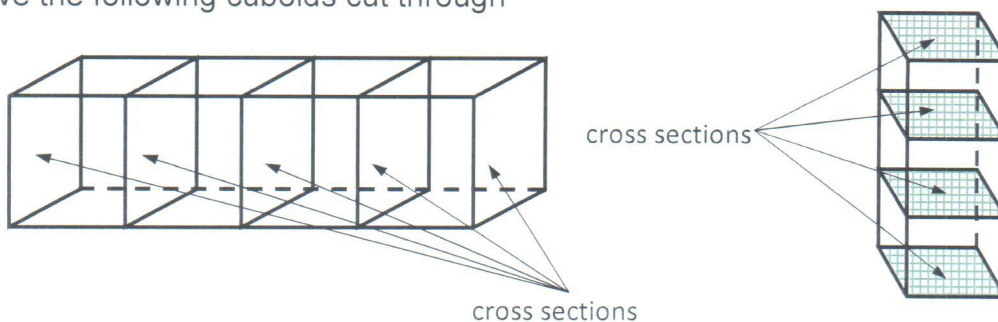


Have you tried measuring them?

Or ever wondered how to measure the surface area of these?

In this lesson, we will focus our discussion in getting the surface area of cubes and cuboids. In Lesson 12, we discussed solving for the area of plane figures. That knowledge is very useful in dealing with solid figures.

Observe the following cuboids cut through

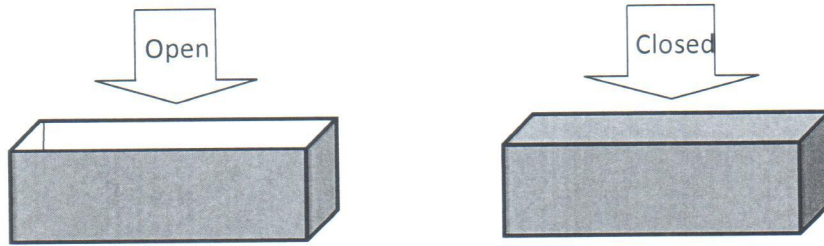


Surface Area refers to the sum of the areas of faces of a solid. In general, the terms **TOTAL SURFACE AREA** refers to the sum of the lateral area and area of the base(s) of closed solid figures. It can be represented by the formula:

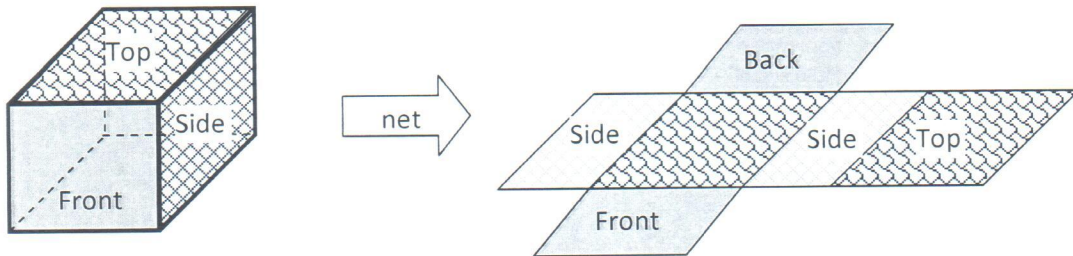
$$SA = L + 2B$$

Where: SA = Surface Area
L = Lateral Area
B = Area of the base

Open and closed cuboids



Surface Area of Closed Cuboids and Cubes (Total Surface Area)

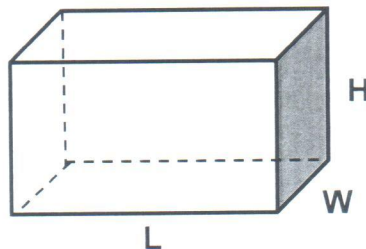


Observe the closed cuboid above. When it is ripped to pieces, you will notice that it is made up of rectangles. Also notice that opposite sides such as the front and back, top and bottom, and the left and right sides are congruent.

With that we can say that the formula for solving the area of a rectangle will be very helpful in this problem.

Legend:

L = length
W = width
H = height



Solving for the areas of
top and bottom parts:

$$2 (L \times W) = 2 LW \quad \text{(Area of the Bases)}$$

front and back parts:

$$2 (L \times H) = 2 LH \quad \left[\begin{array}{c} \text{Lateral Area} \end{array} \right]$$

left and right sides:

$$2 (H \times W) = 2 HW$$

Simplifying it:

$$SA = 2LW + 2LH + 2HW$$

This formula can also be simplified by factoring out 2. $SA = 2(LW + LH + HW)$

Knowing this formula will make solving for the surface area of closed cuboids a lot easier and faster.

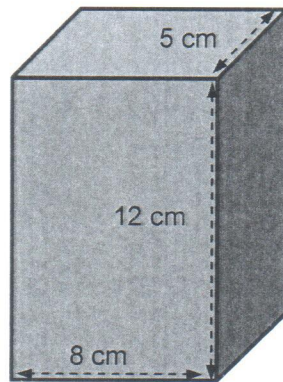
The Surface Area of a Cuboid or a Rectangular Prism is

$$SA = 2 (LW + LH + HW)$$

where: SA = Surface Area ; L = length ; W = width ; H = height

Example 1

Solve for the surface area of the cuboid below.



Solution:

Using the given above where: $L = 8 \text{ cm}$, $W = 5 \text{ cm}$, $H = 12 \text{ cm}$

Substitute in the formula:

$$SA = 2 (LW + LH + HW)$$

$$= 2 [(8\text{cm})(5\text{cm}) + (8\text{cm})(12\text{cm}) + (12\text{cm})(5\text{cm})]$$

Simplify:

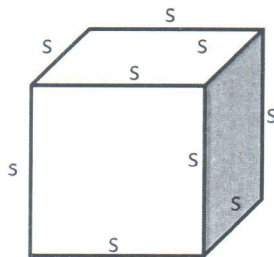
$$= 2 (40\text{cm}^2 + 96\text{cm}^2 + 60\text{cm}^2)$$

$$= 2 (196\text{cm}^2)$$

$$= 392\text{cm}^2$$

Therefore, the total surface area of the cuboid is 392cm^2 .

The same formula can also be used when solving for the total surface area of cubes. But this formula can still be simplified knowing that the cube has equal length of sides.



Using the formula:

$$SA = 2 (LW + LH + HW)$$

but length, width and height are all equal to s

$$SA = 2 (ss + ss + ss) \text{ or } 2(s^2 + s^2 + s^2)$$

This formula can still be simplified

$$SA = 2(3s^2) = 6s^2$$

Hence, the Surface Area of a Cube is

$$SA = 6s^2$$

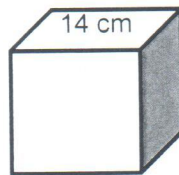
where: **SA** = Surface Area ; **s** = length of one side of the cube

It is a lot simpler, right?

The long method can still be used in case you forgot how you derived the formula. But it is very important if you remember these simpler formulas for ease and speed in computing.

Example 2

Solve for the total surface area of a cube whose side length measures 14 cm.



Solution:

Since the given side length is 14 cm, we let :

$$s = 14 \text{ cm}$$

Using the formula:

$$SA = 6s^2$$

Substitute the given:

$$SA = 6(14 \text{ cm})^2$$

Simplify:

$$= 6(196 \text{ cm}^2)$$

$$= 1176 \text{ cm}^2$$

Therefore, the total surface area of the cube is 1176 cm².

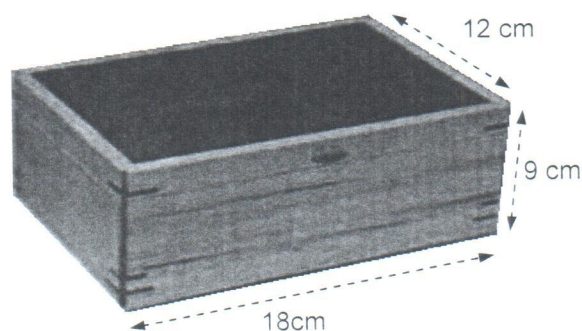
Surface Area of Open Cubes and Cuboids

Since you have already learned how to solve for the surface area of closed cubes and cuboids, it will be very easy to solve for same using open cubes and cuboids. Just remember the formula and subtract the area of the missing part of the cube or cuboid.

Let us try the following example

Example 3

Find the surface area of the open box below.



Solution:

First we get the total surface area of the box using the formula:

$$A_T = 2 (LW + LH + HW)$$

Using the given $L = 18 \text{ cm}$, $W = 12 \text{ cm}$ and $H = 9 \text{ cm}$

Substitute the given in the formula

$$\begin{aligned} SA &= 2 (LW + LH + HW) \\ &= 2 [(18 \text{ cm})(12 \text{ cm}) + (18 \text{ cm})(9 \text{ cm}) + (9 \text{ cm})(12 \text{ cm})] \end{aligned}$$

$$\begin{aligned} \text{Simplify:} \quad &= 2 (216 \text{ cm}^2 + 162 \text{ cm}^2 + 108 \text{ cm}^2) \\ &= 2 (486 \text{ cm}^2) \\ &= 972 \text{ cm}^2 \end{aligned}$$

Next, we get the area of the missing part

Since the top part is missing its area is represented by

$$A = L \times W$$

Substitute:

$$A = (18 \text{ cm})(12 \text{ cm})$$

Simplify:

$$A = 216 \text{ cm}^2$$

Lastly, we subtract the area of the missing part from the total surface area of the box

$$\begin{aligned} \text{Surface area of the open box} &= \text{total surface area} - \text{area of the missing part (top)} \\ &= 972 \text{ cm}^2 - 216 \text{ cm}^2 \\ &= \mathbf{756 \text{ cm}^2} \end{aligned}$$

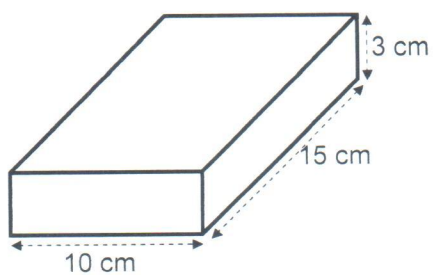
Therefore, the surface area of the open box is 756 cm^2 .

NOW DO PRACTICE EXERCISE 13

**Practice Exercise 13**

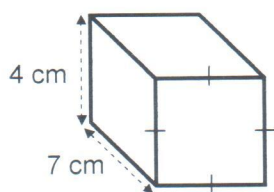
Find the surface area of the following cubes and cuboids:

1) a)



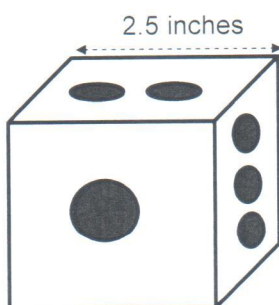
Answer: _____

b)



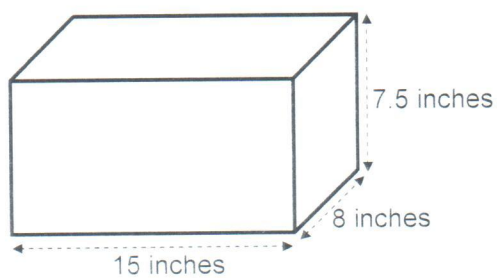
Answer: _____

c)



Answer: _____

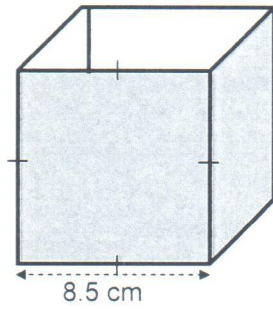
d)



Answer: _____

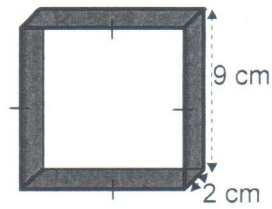
2.

a) .



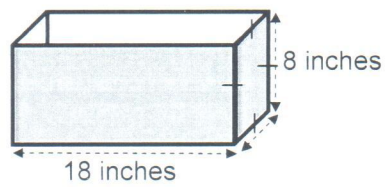
Answer: _____

b)



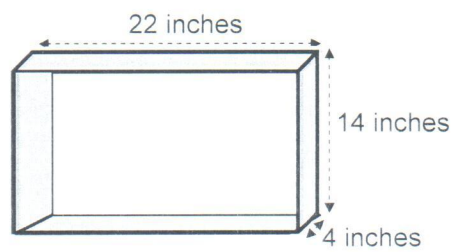
Answer: _____

c)



Answer: _____

d)



Answer: _____

3. Solve the following problems. Show your solution (working out):
- a) A rectangular swimming pool has the following dimensions: 20 ft x 6 ft x 12 ft. The owner would like to use green tiles on the sides and white one on the floor. If each tile measures 2 ft x 2 ft each, how many green tiles are needed? How many white tiles are needed?
- b) Armella would like to recycle 20 pieces of used cube carton boxes. The top and bottom parts of the boxes are already broken so she decided to remove them instead. If one side of the box measures 0.5 meters, how many cartons did she recycle?
- c) Ignatius would like to paint his room using the latest paint in town. He was told by his dad that the following are the dimensions of his room: height 5.5 m, width 8 m, length 10 m. If one can of paint cover an area of 20 m^2 , how many cans of paint will Ignatius buy?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
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Lesson 14: Surface Area of Triangular and Trapezoidal Prisms



You have learnt about surface area of cube and cuboids in the previous lesson.



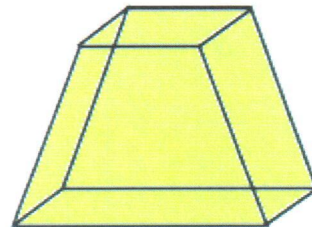
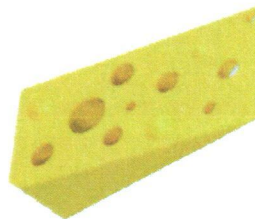
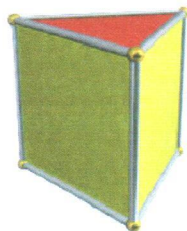
In this lesson you will:

- identify and calculate the surface area of triangular and trapezium prisms

In Lesson 13, you have learned that the Total Surface Area of cubes and cuboids could be determined by adding the areas of all their faces. You also learned to derive formulas to make computation a lot easier.

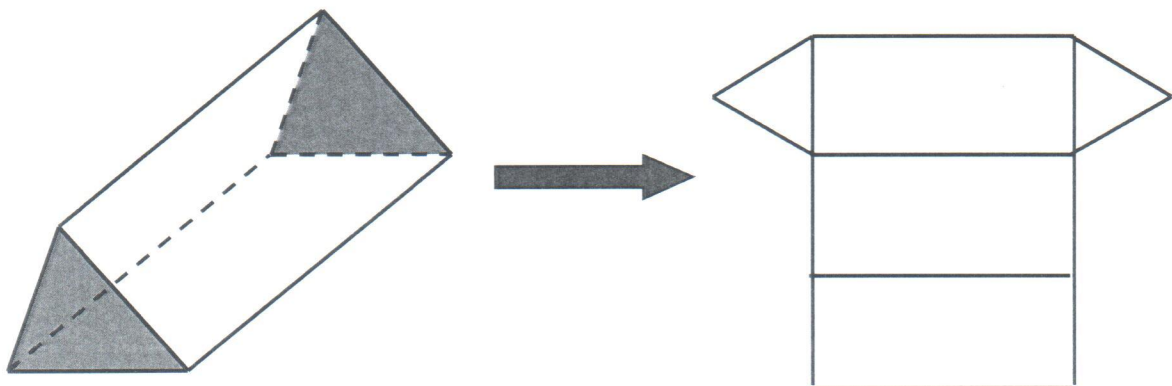
In this lesson you will learn to solve for the surface areas of triangular prism and trapezium prisms.

Observe the objects below:

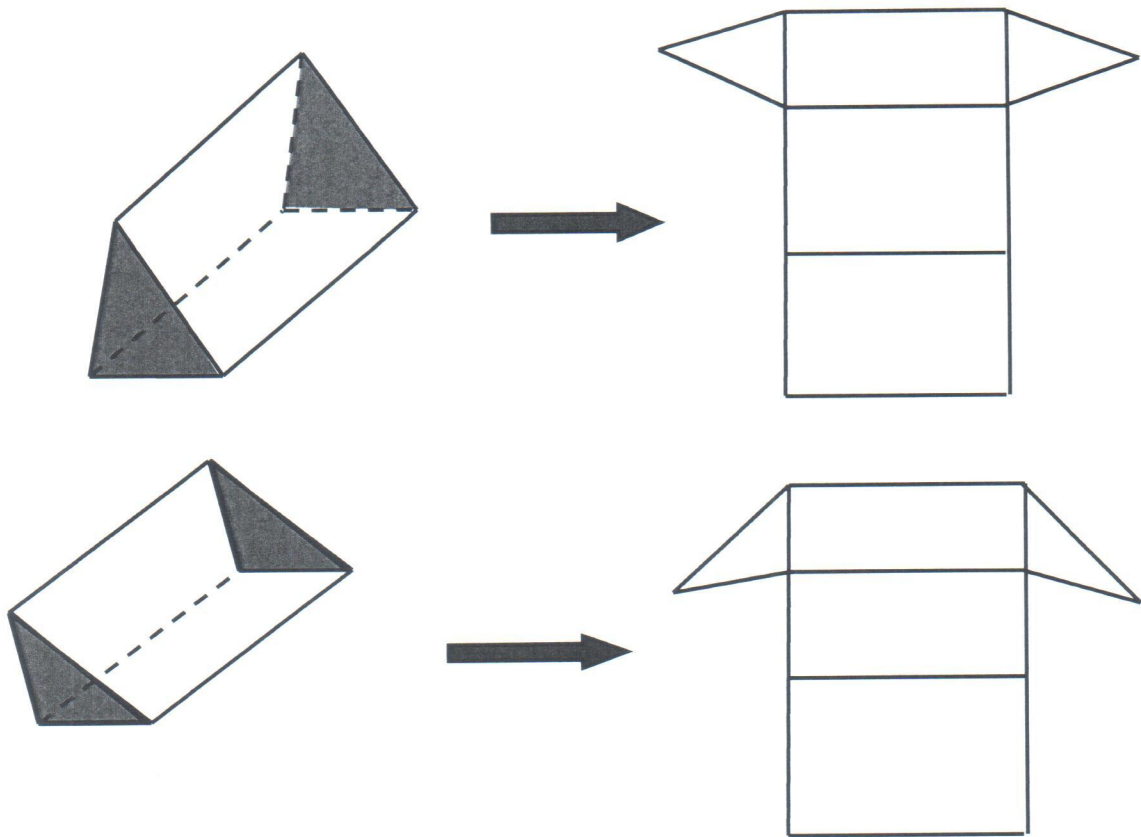


Did you notice that their faces and bases are polygons?

Let us take a closer look at the net of this triangular prism.



Notice that the triangular prism has 5 faces. Three rectangular lateral faces and 2 triangular bases. Sometimes the lateral bases are congruent when the triangular bases are equilateral triangles just like the figure above. But if the triangular bases are scalene or isosceles, expect that the lateral faces may have different areas. The following triangular prisms whose bases are isosceles and scalene show the differences of the lateral faces when the bases are not equilateral.



That is why there is no single formula for surface area which could be applied to all triangular prisms. Instead, the general rule of adding all the areas of each face may be applied.

Surface Area of Triangular Prism is the sum of the area of the 3 lateral faces and 2 bases. It can be denoted as:

$$SA = A_{b1} + A_{b2} + A_3 + A_4 + A_5$$

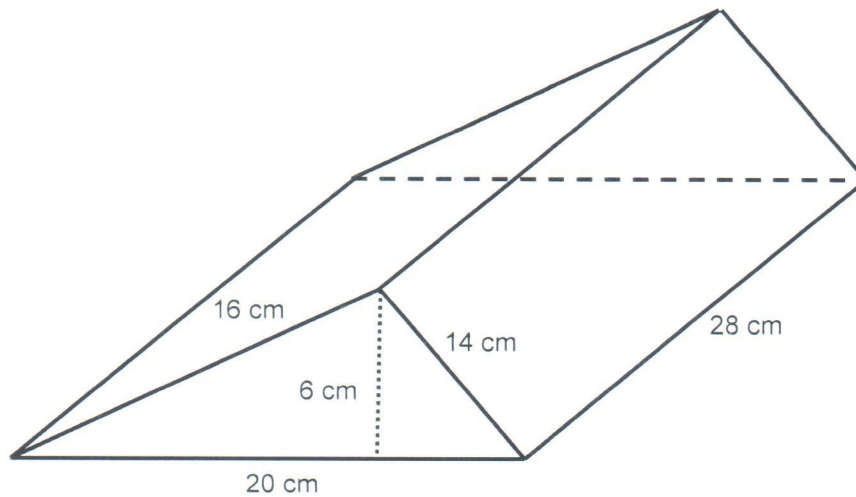
or simplified as

$$SA = 2(A_b) + A_3 + A_4 + A_5$$

Since the areas of the two bases are always congruent.

Example 1

Find the surface area of the triangular prism drawn below.



Note that the bases are triangles; therefore we recall the formula in solving for the area of a triangle $A_b = \frac{1}{2}bh$ and the formula in solving for the area of rectangle $A = lw$.

First let us find the areas of the bases and the faces.

$$A_{b1} = \frac{1}{2} \times \text{base} \times \text{height} \quad A_{b1} = \frac{1}{2} \times 20 \text{ cm} \times 6 \text{ cm} = 60 \text{ cm}^2$$

$$A_{b2} = \frac{1}{2} \times \text{base} \times \text{height} \quad A_{b2} = \frac{1}{2} \times 20 \text{ cm} \times 6 \text{ cm} = 60 \text{ cm}^2$$

$$A_3 = \text{length} \times \text{width} \quad A_3 = 14 \text{ cm} \times 28 \text{ cm} = 392 \text{ cm}^2$$

$$A_4 = \text{length} \times \text{width} \quad A_4 = 16 \text{ cm} \times 28 \text{ cm} = 448 \text{ cm}^2$$

$$A_5 = \text{length} \times \text{width} \quad A_5 = 20 \text{ cm} \times 28 \text{ cm} = 560 \text{ cm}^2$$

Next, get the sum of all areas.

$$SA = A_{b1} + A_{b2} + A_3 + A_4 + A_5$$

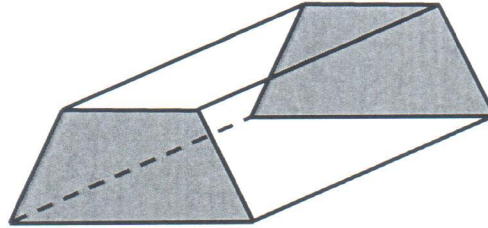
$$SA = 60 \text{ cm}^2 + 60 \text{ cm}^2 + 392 \text{ cm}^2 + 448 \text{ cm}^2 + 560 \text{ cm}^2$$

$$SA = 1520 \text{ cm}^2$$

Therefore, the total surface area of the triangular prism drawn above is 1520 cm^2 .

Trapezoidal Prism

A **trapezoidal prism** also known as trapezoidal prism has six faces. Its two bases are trapezium while the 4 lateral faces are rectangles. Just like the triangular prism, the general rule of adding the areas of all faces applies.



Surface Area of Trapezoidal Prism is the sum of the area of the 3 lateral faces and 2 bases. It can be denoted as:

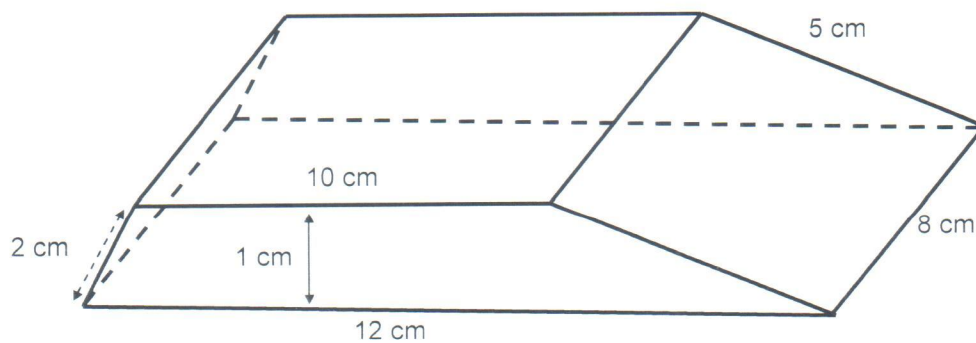
$$SA = A_{b1} + A_{b2} + A_3 + A_4 + A_5 + A_6$$

or simplified as

$$SA = 2(A_b) + A_3 + A_4 + A_5 + A_6$$

Since the areas of the two bases are always congruent.

Example 2



Note that the bases are trapeziums; therefore we recall the formula in solving for the area of a trapezium $A_b = \frac{1}{2}(b_1 + b_2)(h)$ and the formula in solving for the area of rectangle $A = LW$.

First let us find the areas of the bases and the faces.

$$(a) \quad A_{b1} = \frac{1}{2} \times (\text{upper base} + \text{lower base}) \times \text{height}$$

$$A_{b1} = \frac{1}{2} \times (10 \text{ cm} + 12 \text{ cm})(1 \text{ cm}) = 11 \text{ cm}^2$$

$$(b) \quad A_{b2} = \frac{1}{2} \times (\text{upper base} + \text{lower base}) \times \text{height}$$

$$A_{b2} = \frac{1}{2} \times (10 + 12) \times 1 = 11 \text{ cm}^2$$

$$(c) \quad A_3 = \text{length} \times \text{width} \quad A_3 = 5 \text{ cm} \times 8 \text{ cm} = 40 \text{ cm}^2$$

$$(d) \quad A_4 = \text{length} \times \text{width} \quad A_4 = 10 \text{ cm} \times 8 \text{ cm} = 80 \text{ cm}^2$$

$$(e) \quad A_5 = \text{length} \times \text{width} \quad A_5 = 2 \text{ cm} \times 8 \text{ cm} = 16 \text{ cm}^2$$

$$(f) \quad A_6 = \text{length} \times \text{width} \quad A_6 = 12 \text{ cm} \times 8 \text{ cm} = 96 \text{ cm}^2$$

Next, get the sum of all areas.

$$SA = A_{b1} + A_{b2} + A_3 + A_4 + A_5 + A_6$$

$$SA = 11 \text{ cm}^2 + 11 \text{ cm}^2 + 40 \text{ cm}^2 + 80 \text{ cm}^2 + 16 \text{ cm}^2 + 96 \text{ cm}^2$$

$$SA = 254 \text{ cm}^2$$

Therefore, the total surface area of the trapezium prism is 254 cm^2 .

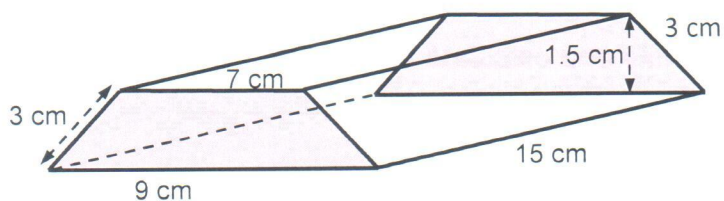
NOW DO PRACTICE EXERCISE 14



Practice Exercise 14

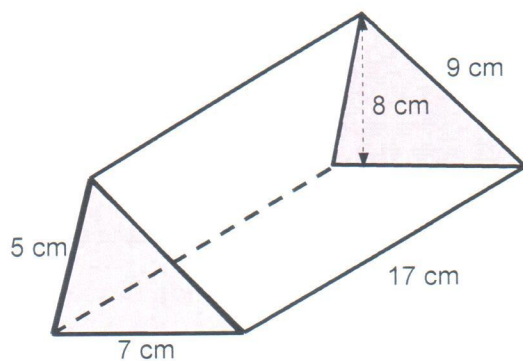
Find the surface area of the following solids

1.



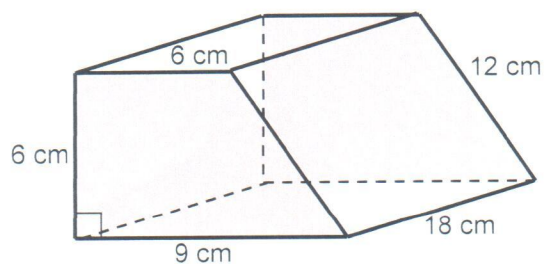
Answer: _____

2.



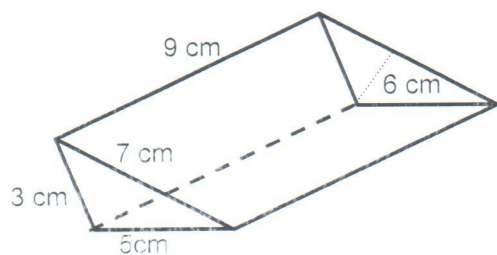
Answer: _____

3.



Answer: _____

4.



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 15: Surface Area of Pyramids



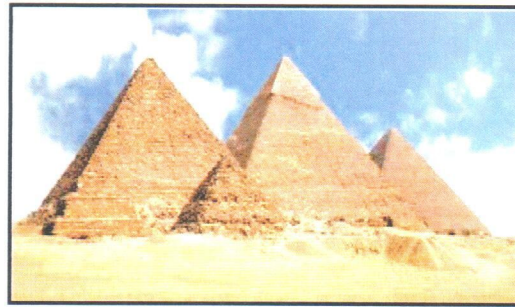
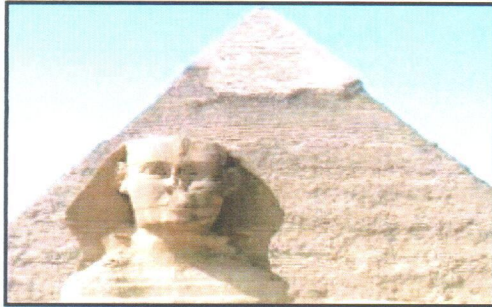
You have learned about surface area in the previous lessons.



In this lesson, you will:

- identify and calculate the surface area of pyramids.

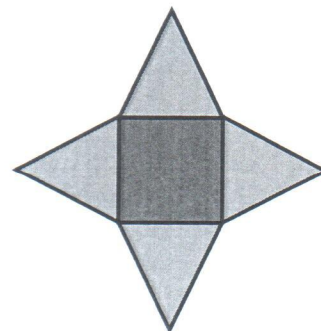
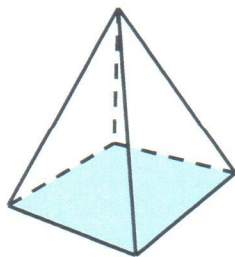
Are you familiar with the following figures?



The above figures show the great pyramids of Egypt. These pyramids stood strong in the desert for thousands of years. These structures were also used to bury or mummify their dead loved ones.

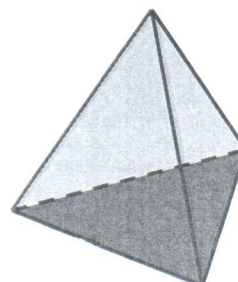
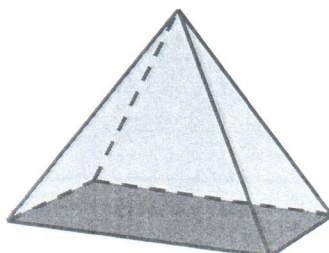
Aside from the interesting facts in history about the pyramids, there are other useful and interesting things we will discover about them in this lesson.

Observe the net of the pyramid.



What did you notice about its faces?
How about its base?

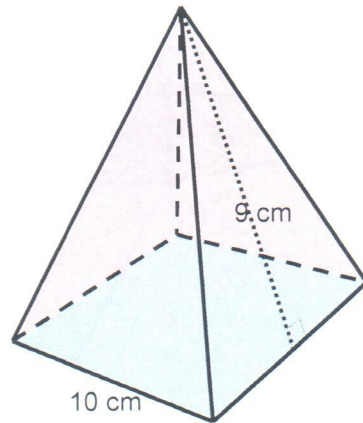
Aside from the square base of the pyramid shown above, its base could also be a rectangle or a triangle (tetrahedron) as shown below;



Since we know that the bases of pyramids vary from square, rectangle and another triangle while its faces are made up of triangles, we can now say that the formula for its surface area depends on the kind of base a pyramid has.

Example 1

Find the surface area of the square based pyramid drawn below:



Note that base is a square, therefore we recall the formula in solving for the area of a square $s = s^2$, and since the faces are triangles, we also use the formula $A = \frac{1}{2}bh$.

First let us find the areas of the square base and the triangular faces.

$$A_{\text{base}} = s^2 \qquad A_3 = (10 \text{ cm})^2 \qquad = \qquad 100 \text{ cm}^2$$

$$A_{\text{faces}} = \frac{1}{2} \times \text{base} \times \text{height} \qquad A_{b1} = \frac{1}{2} \times 10 \text{ cm} \times 9 \text{ cm} \qquad = \qquad 45 \text{ cm}^2$$

Since the base is square, we have to note that all of the four faces of the square based pyramid are congruent.

Next, get the sum of all areas.

$$SA = A_b + A_1 + A_2 + A_3 + A_4$$

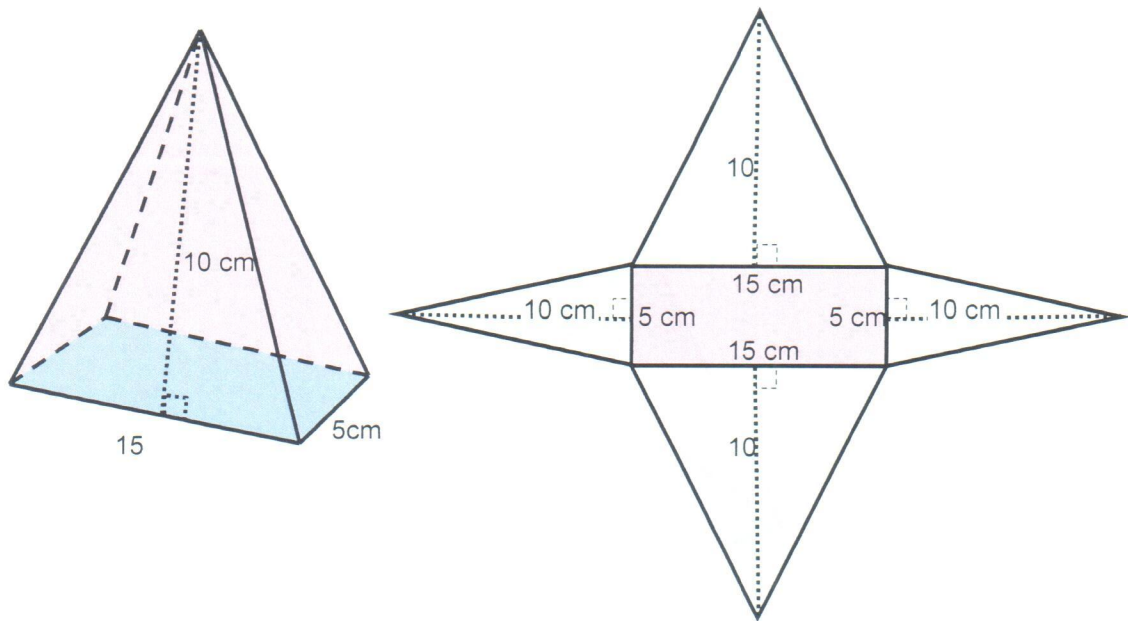
$$SA = 100 \text{ cm}^2 + 45 \text{ cm}^2 + 45 \text{ cm}^2 + 45 \text{ cm}^2 + 45 \text{ cm}^2$$

$$SA = 280 \text{ cm}^2$$

Therefore, the total surface area of the square based pyramid is 280 cm^2 .

Example 2

Find the surface area of the rectangular based pyramid drawn below:



Note that base is a rectangle, therefore we recall the formula in solving for the area of a rectangle: $A = LW$, and since the faces are triangles, we also use the formula for the area of a triangle, $A = \frac{1}{2}bh$.

The net of the rectangular based pyramid shows that the front and back faces are congruent, while the left and right side faces are also congruent.

Let us find the areas of the rectangle base and the triangular faces.

$$A_{\text{base}} = LW \qquad A_{\text{base}} = 15 \times 5 \qquad = \qquad 75 \text{ cm}^2$$

Get the area of the front and back faces

$$A_{\text{faces}} = \frac{1}{2} \times \text{base} \times \text{height} \qquad A_{b1} = \frac{1}{2} \times 15 \text{ cm} \times 10 \text{ cm} \qquad = \qquad 75 \text{ cm}^2$$

Get the area of the left and right side faces

$$A_{\text{faces}} = \frac{1}{2} \times \text{base} \times \text{height} \qquad A_{b1} = \frac{1}{2} \times 5 \text{ cm} \times 10 \text{ cm} \qquad = \qquad 25 \text{ cm}^2$$

Next, get the sum of all areas.

$$SA = A_b + A_1 + A_2 + A_3 + A_4$$

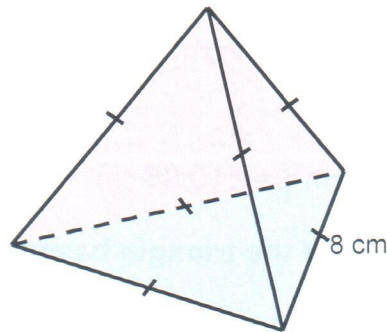
$$SA = 75 \text{ cm}^2 + 75 \text{ cm}^2 + 75 \text{ cm}^2 + 25 \text{ cm}^2 + 25 \text{ cm}^2$$

$$SA = 275 \text{ cm}^2$$

Therefore, the total surface area of the square based pyramid is 275 cm^2 .

Example 3

Find the surface area of the tetrahedron below whose lengths of sides are congruent:

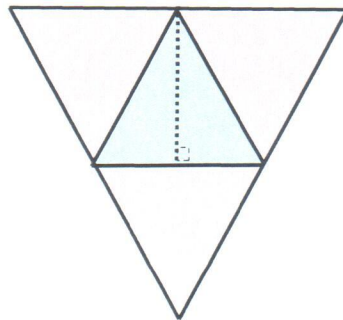


The given is just a length of one side of the base but since the problem states that all sides are congruent, it also means that the sides of each face measure 8 cm.

We also have to note that the base and all faces are triangles, so we will only use one formula, that is the formula in getting the area of a triangle which is $A = \frac{1}{2}bh$.

But how can we solve for the area of the triangles if the only given is a measure of the base?

Observe the net of the tetrahedron, since the base and each face are equilateral triangles, we can solve for the height of one triangle using the Pythagoras' Theorem.



Recall that in your previous Lessons about triangles, its height or altitude is the line drawn from one vertex perpendicular to the opposite side. In the case of an equilateral triangle, the height also bisects (divides) the opposite side into two congruent parts. Since the measure of each side is 8 cm, half of it is 4 cm.

Now let us solve for the height of the triangle:

$$h = \sqrt{8^2 - 4^2}$$

$$h = \sqrt{64 - 16}$$

$$h = \sqrt{48}$$

$$h = 6.93 \text{ cm}$$

Now we are ready to solve for the area of the triangle

$$A_{\text{base}} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A_{b1} = \frac{1}{2} \times 8 \text{ cm} \times 6.93 \text{ cm} = 27.72 \text{ cm}^2$$

Next, get the sum of all areas. Since the base and faces are all congruent, we can say that

$$SA = A_b + A_1 + A_2 + A_3$$

$$SA = 27.72 \text{ cm}^2 + 27.72\text{cm}^2 + 27.72\text{cm}^2 + 27.72 \text{ cm}^2$$

$$SA = 110.88 \text{ cm}^2$$

Or simply say, $SA = 4 (27.72\text{cm}^2) = 110.88 \text{ cm}^2$

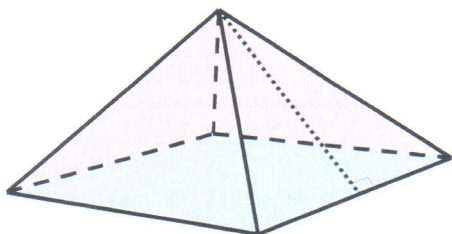
Therefore, the total surface area of the triangle based pyramid or tetrahedron is 110.88 cm^2 .

NOW DO PRACTICE EXERCISE 15

**Practice Exercise 15**

1. Draw the nets of the following pyramids and label them properly, then solve for their total surface areas:

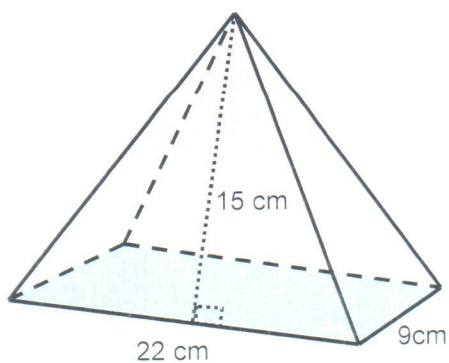
a)



Draw the net here:

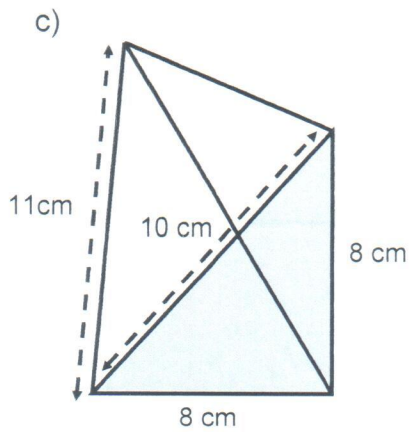
Total Surface Area: _____

b)



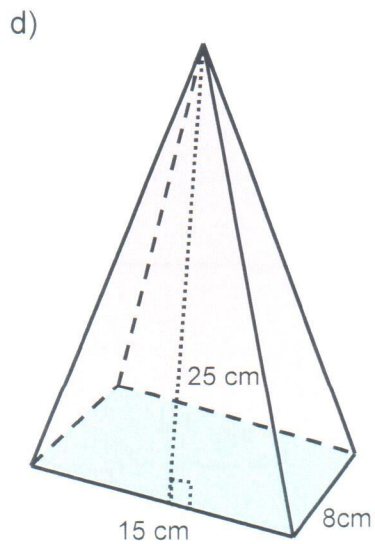
Draw the net here:

Total Surface Area: _____



Draw the net here:

Total Surface Area: _____



Draw the net here:

Total Surface Area: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 16: Surface Area of Cylinders



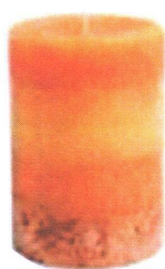
You have learned something about surface area of different solid figures in the previous lessons.



In this lesson, you will:

- describe the nets of cylinders and find their surface area.

Familiar with the following objects?

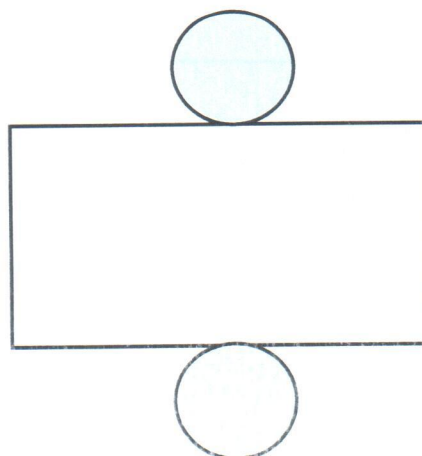
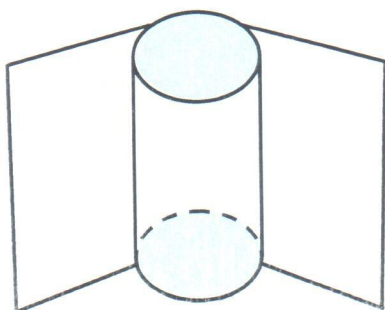


What did you notice about them?

Every day we find these solid figures around us. In your previous Mathematics you were already introduced with the cylinders, right?

In this lesson, our discussion will be focused in solving for the surface area of the cylinder.

Observe the net of the cylinder.



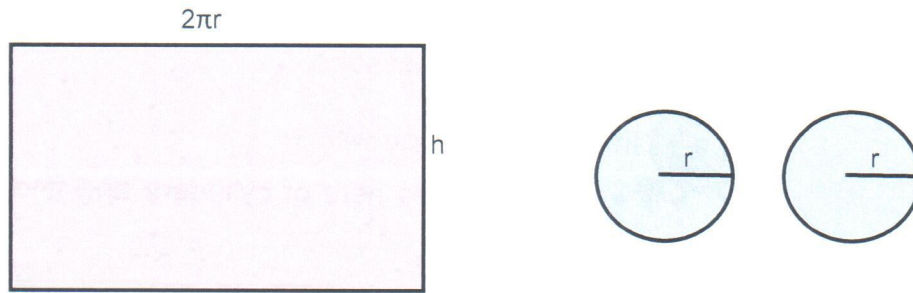
What did you notice?

At first the cylinder may look complicated right?

But if you take a closer look at its net, you will notice that its bases are two congruent circles its curved face is actually a rectangle.

What did you notice about the length of the rectangle?

Great observation if you say that it is equal to the distance around the circle. And this distance is known as the circumference of the circle. Therefore we can label the rectangle as:



Now we recall the formula in solving for the area of a rectangle:

$$A = \text{length} \times \text{height}$$

Since the length is equal to $2\pi r$, we now say that $A = (2\pi r) (\text{height})$

Now we solve for the area of the bases, we recall the formula in solving for the area of a circle:

$$A = \pi r^2$$

Since the bases are congruent, we just multiply the area by 2.

Combining all these ideas we now come up with a simplified way of solving for the surface area of cylinder.

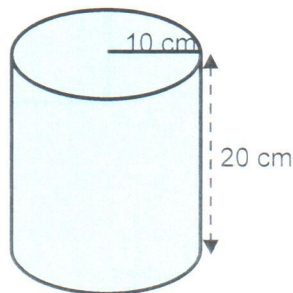
$$\text{Surface area} = 2\pi rh + 2\pi r^2$$

The Total Surface Area of a Closed Cylinder refers to the Area of the rectangular curved face plus the area of the two circular bases. In symbol we say:

$$\text{TSA} = 2\pi rh + 2\pi r^2$$

Example 1

Find the surface area of a cylindrical container drawn below:



Given:

$$r = 10 \text{ cm}, h = 20 \text{ cm}$$

Use the formula:

$$\text{SA} = 2\pi rh + 2\pi r^2$$

Substitute given in the formula:

$$TSA = 2(3.14)(10 \text{ cm})(20\text{cm}) + 2(3.14)(10)^2$$

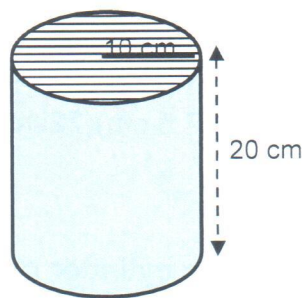
Simplify:

$$\begin{aligned} TSA &= 2(3.14)(10 \text{ cm})(20\text{cm}) + 2(3.14)(10)^2 \\ &= 1256 \text{ cm}^2 + 628 \text{ cm}^2 \\ &= 1884 \text{ cm}^2 \end{aligned}$$

Therefore the total surface area of the cylinder is 1884 cm^2 .

Example 2

What is the surface area of the same can if its upper base is taken out? (open can)



Given: $r = 10 \text{ cm}$, $h = 20 \text{ cm}$

Since the can is open on its top base, we modify the formula by subtracting one area of the base, so instead of $2\pi r^2$, we will only use πr^2 in the formula.

Use the formula: $SA = 2\pi rh + \pi r^2$

Substitute given in the formula:

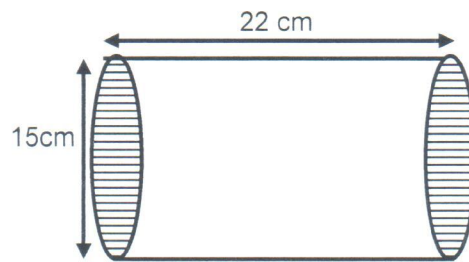
$$SA = 2(3.14)(10 \text{ cm})(20\text{cm}) + (3.14)(10)^2$$

$$\begin{aligned} \text{Simplify: } SA &= 2(3.14)(10 \text{ cm})(20\text{cm}) + (3.14)(10)^2 \\ &= 1256 \text{ cm}^2 + 314 \text{ cm}^2 \\ &= 1570 \text{ cm}^2 \end{aligned}$$

Therefore the surface area of the cylinder open on top is 1570 cm^2 .

Example 3

What is the surface area of the open cylindrical can below?



Given: **$d = 15 \text{ cm}$** , since r is half of the diameter, **$r = 7.5 \text{ cm}$** **$h = 20 \text{ cm}$**

Since the can is open on both ends, we modify the formula by subtracting the area of the bases in $TSA = 2\pi rh + 2\pi r^2$ which gives us $SA = 2\pi rh$.

Use the formula: $SA = 2\pi rh$

Substitute given in the formula:

$$SA = 2(3.14)(7.5 \text{ cm})(22\text{cm})$$

Simplify: $= 1036.2 \text{ cm}^2$

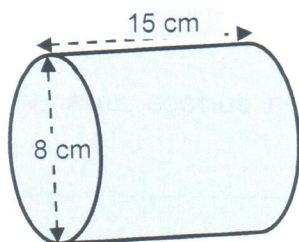
Therefore the surface area of the cylinder open on both ends is 1036.2 cm^2 .

NOW DO PRACTICE EXERCISE 16

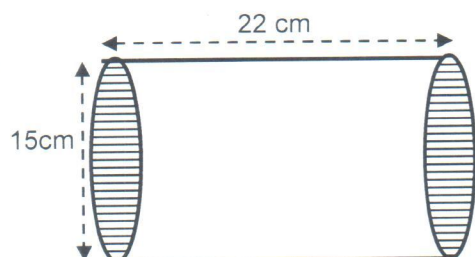
**Practice Exercise 16**

Solve for the surface area of the following cylinders:

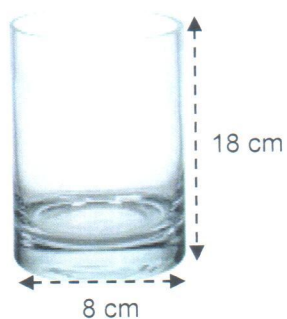
1)



2)



3)



4)



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 17: Surface Area and Problem Solving



You have learnt about solving for the surface areas of different solid figures in the previous lessons.



In this lesson, you will:

- solve general problems on surface area of various solids.

Since you already made yourself familiar with the different solid figures and you were able to devise simpler ways and formulas to make the computation of surface area with speed and accuracy, you are now ready to solve problems involving surface area.

But before that, please take note of the following steps which will help you facilitate solving with ease and accuracy:

1. Make a clear diagram and label them properly.
2. Take note of the kind of solid figure you are dealing with, count the number of faces and identify the base(s).
3. Calculate the area of each face. Make sure that no face has been left out.
4. Get the sum of all areas of all faces.
5. Do not forget to affix the unit of measurement used. Make sure that the unit is squared.

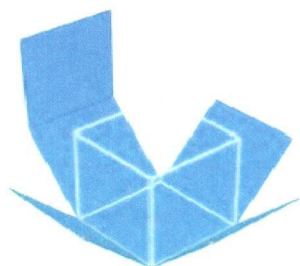
Having all these in mind, you are now ready to apply what you have learned in the previous lessons.

Example 1

For their school project on solid figures Marie made a cube frame out of recycled wires, she would like to wrap it with a special paper to make it more appealing. How much paper does she need if one side of the cube measures 18 cm?

Solution:

Since the problem involves a cube, and Marie would like to wrap the cube, the problem suggest that we get the surface area and use the formula $SA = 6s^2$.

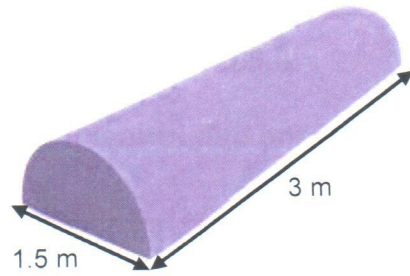


Given:	$s = 18 \text{ cm}$
Formula:	$SA = 6s^2$
Substitute:	$SA = 6(18\text{cm})^2$
Simplify:	$SA = 6(324\text{cm}^2)$
	$SA = 1\,944\text{cm}^2$

Therefore, Marie needs a special paper with an area of at least 1944 cm^2 .

Example 2

What is the surface area of the log drawn below?



Given:

$$d = 1.5 \text{ m, therefore } r = 0.75 \text{ m, } h = 3 \text{ m}$$

Use the formula:

$$SA = 2\pi rh + 2\pi r^2$$



But the log is cut into halves, and we only need to get the surface area of half of it.

We can do 2 things, one is to divide the formula by 2 before we solve, and the other is to solve for the surface area of the whole log and then divide the quotient by 2.

Let us try the first one by modifying the formula:

$$SA = 2\pi rh + 2\pi r^2$$

Multiply $\frac{1}{2}$ to the formula:

$$SA = \frac{1}{2}(2\pi rh + 2\pi r^2)$$

$$\text{TSA of half the log} = \pi rh + \pi r^2$$

Substitute given in the formula:

$$SA = (3.14)(0.75\text{m})(3\text{m}) + (3.14)(0.75\text{m})^2$$

$$SA = (3.14)(0.75\text{m})(3\text{m}) + (3.14)(0.5625\text{m}^2)$$

Simplify:

$$\begin{aligned} SA &= 7.065 \text{ m}^2 + 1.76625 \text{ m}^2 \\ &= 8.83125 \text{ m}^2 \end{aligned}$$

Now let us try the second method:

Use the formula:

$$SA = 2\pi rh + \pi r^2$$

Substitute given in the formula:

$$SA = 2(3.14)(0.75\text{m})(3\text{m}) + 2(3.14)(0.75)^2$$

Simplify:

$$\begin{aligned}
 SA &= 2(3.14)(0.75\text{m})(3\text{cm}) + 2(3.14)(0.5625\text{m}^2) \\
 &= 14.13\text{m}^2 + 3.5325\text{m}^2 \\
 &= 17.6625\text{m}^2
 \end{aligned}$$

Divide the SA by 2: $17.6625\text{m}^2 \div 2 = 8.83125\text{m}^2$

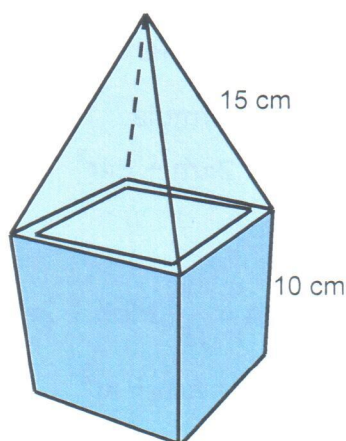
Therefore the surface area of half of the cylindrical log is 8.83 m^2 .



Two methods were presented to you; you choose the best and suitable method that works for you. You can also challenge yourself to derive a formula based on the given.

Example 3

Peter made a candle holder using fibre glass whose base is a hollow cube and it is topped with a pyramid as shown below. His friend Joe asked him how much materials he used. Help Peter answer Joe's question.



Looking at the figure, there are 2 solids involved. The base is a cube and the upper part is a pyramid. We also need to consider that both solids are hollow inside because Peter made a candle holder.

Let us start solving for the surface area of the cube. Since the cube is open on top and bottom, we will only consider its 4 faces. Therefore we use the formula $SA = 4s^2$.

Given: $s = 10 \text{ cm}$

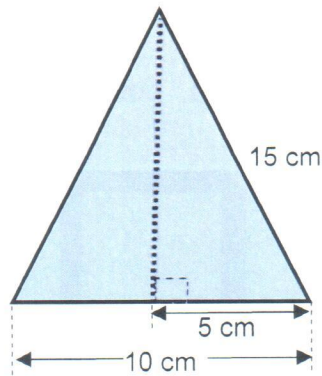
Formula: $SA = 4s^2$

Substitute: $SA = 4(10\text{cm})^2$

Simplify: $SA = 4(100\text{cm}^2)$

$SA_{(\text{Cube})} = 400 \text{ cm}^2$

Now we solve for the square based pyramid. Notice too that the pyramid is open at its base, therefore we will not include the base in the computation of the surface area. Looking at one face of the pyramid, we have the following given:



We need to solve for the height of one of its faces before we can proceed solving for the surface area.

Using the Pythagoras' Theorem, we solve for the height of one of the triangular faces.

$$h = \sqrt{15^2 - 5^2}$$

$$h = \sqrt{225 - 25}$$

$$h = \sqrt{200}$$

$$h = 14.14 \text{ cm}$$

Now we are ready to solve for the area of the triangle.

Using the following given:

$$b = 10 \text{ cm}$$

$$h = 14.14 \text{ cm}$$

$$A_{\text{face}} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A_{\text{face}} = \frac{1}{2} \times 10 \text{ cm} \times 14.14 \text{ cm} = 70.7 \text{ cm}^2$$

Since we will only consider the 4 congruent triangular faces, we multiply the area of one triangle by 4:

$$SA_{(\text{Pyramid})} = 4(70.70 \text{ cm}^2) = 282.8 \text{ cm}^2$$

Next, get the total of the surface areas (TSA) of the two solids

$$TSA = SA_{(\text{cube})} + SA_{(\text{pyramid})}$$

$$TSA = 400 \text{ cm}^2 + 282.8 \text{ cm}^2$$

$$TSA = 682.8 \text{ cm}^2$$

Therefore, Peter used a total of 682.8 cm^2 of materials to cover the candle holder he made.

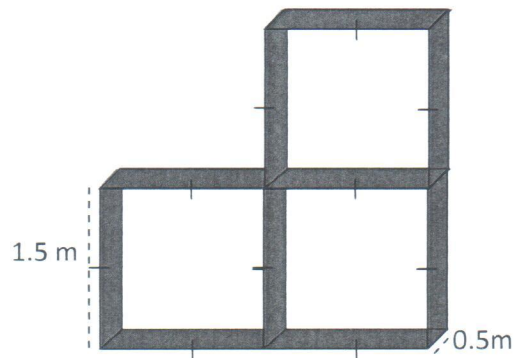
NOW DO PRACTICE EXERCISE 17



Practice Exercise 17

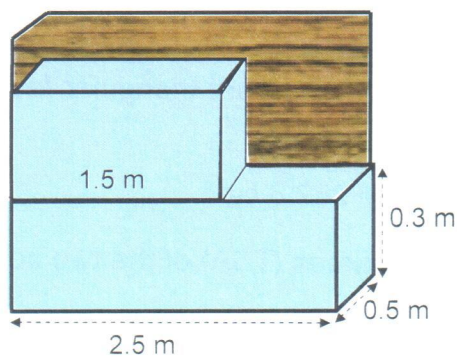
Solve the surface area given the following conditions:

1. Solve for the external surface area, bottom parts and attached surface not included.



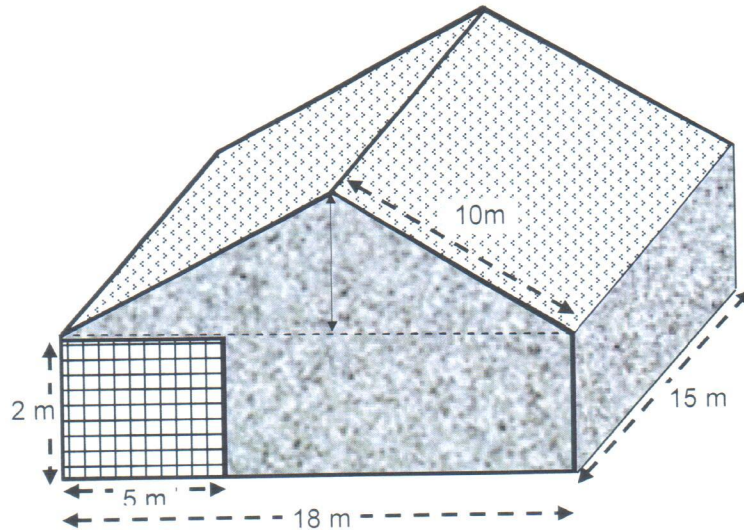
Answer: _____

2. A two-step stairs was attached on a wooden wall. A carpenter would like to put a parquet (wooden floor board) on it. How much of the materials will he need?



Answer: _____

3. Use the figure below to answer the questions that follow:



a) What is the area of the gate?

Answer: _____

b) What is the area of the roof?

Answer: _____

c) What is the floor area of the warehouse?

Answer: _____

d) What is the height of the ceiling from the floor?

Answer: _____

e) What is the surface area of the walls (including the back and side walls but not the gate)?

Answer: _____

f) What is the total surface area of the warehouse excluding the floor?

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

TOPIC 3: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- Composite Shapes are plane figures which are irregular in shape. Basically the composite shape is composed of a combination of two or more polygons or a circle.
- Surface Area refers to the sum of the areas of faces of a solid. In general, the terms TOTAL SURFACE AREA refers to the sum of the lateral area and area of the base(s) of closed solid figures.
- The Surface Area of a Cuboid or a Rectangular Prism is:

$$SA = 2(LW + LH + HW)$$

where: SA = Surface Area ; l = length ; w = width ; h = height

- The Surface Area of a Cube is: $SA = 6s^2$

where: SA = Surface Area ; s = length of one side of the cube

- Surface Area of Triangular Prism is the sum of the area of the 3 lateral faces and 2 bases. It can be denoted as $SA = A_{b1} + A_{b2} + A_3 + A_4 + A_5$ or simplified as

$$SA = 2(A_b) + A_3 + A_4 + A_5.$$

- Surface Area of Trapezium Prism is the sum of the area of the 3 lateral faces and 2 bases. It can be denoted as $SA = A_{b1} + A_{b2} + A_3 + A_4 + A_5 + A_6$ or simplified as:

$$SA = 2(A_b) + A_3 + A_4 + A_5 + A_6$$

- The Total Surface Area of a Closed Cylinder refers to the Area of the rectangular curved face plus the area of the two circular bases. In symbol we say:

$$TSA = 2\pi rh + 2\pi r^2$$

REVISE LESSONS 12 - 17. THEN DO TOPIC TEST 3 IN ASSIGNMENT BOOK 6.

ANSWERS TO PRACTICE EXERCISES 12 - 17**Practice Exercise 12**

1. 232.5 cm^2 2. 379.56 cm^2 3. 30.27 m^2 4. 262.97 m^2

Practice Exercise 13

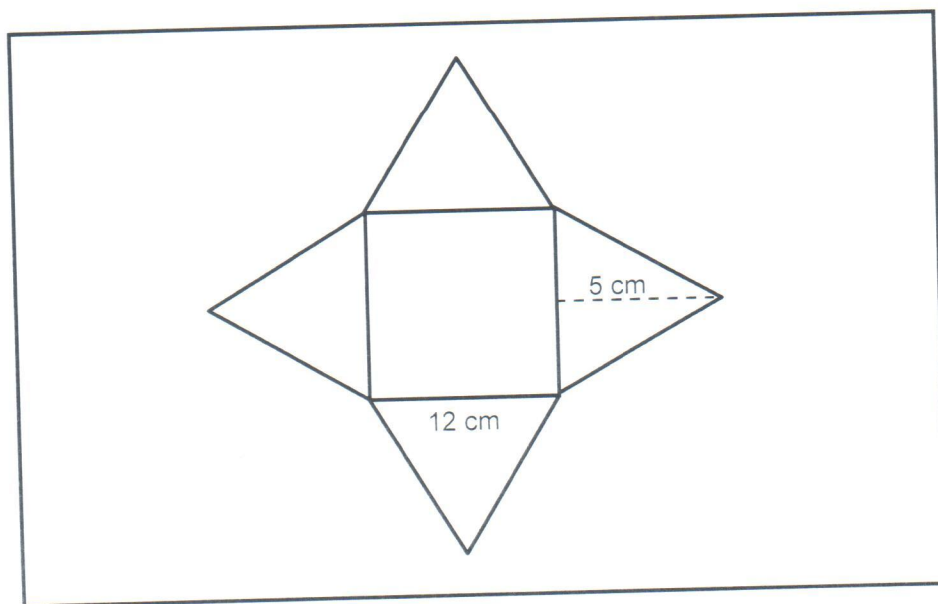
1. a) 450 cm^2 b) 344 cm^2 c) 37.5 sq. inches d) 585 sq. inches
2. a) 361.25 cm^2 b) 560 sq. inches c) 596 sq. inches
3. a). 96 green tiles & 60 white tiles b) 13 cartons c) 17 cans of paint

Practice Exercise 14

1. 304.5 cm^2
2. 4814 cm^2
3. 360 cm^2
4. 177 cm^2

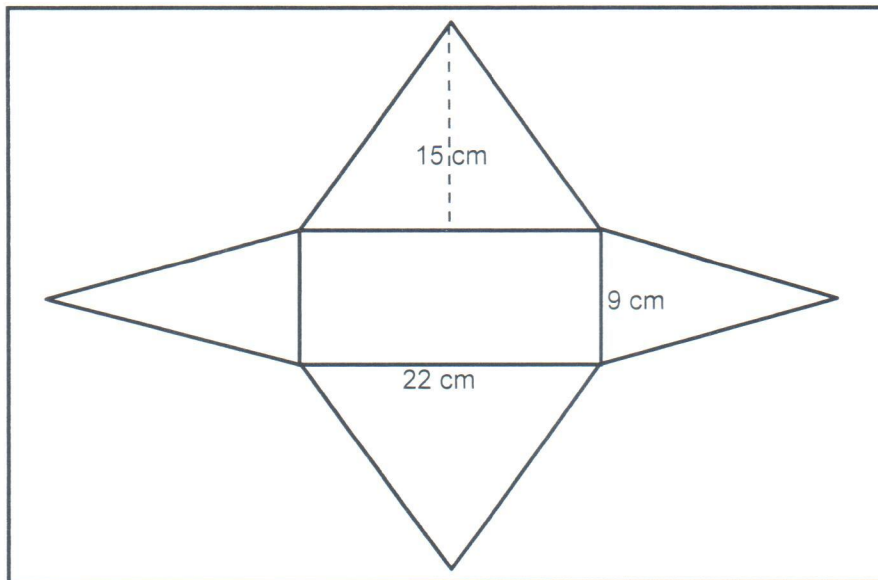
Practice Exercise 15

1. a)



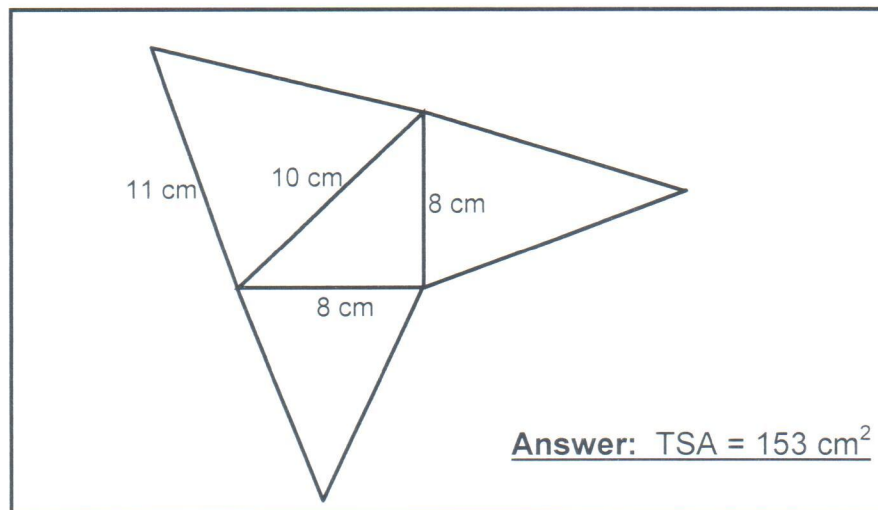
Answer: TSA = 264 cm^2

b)



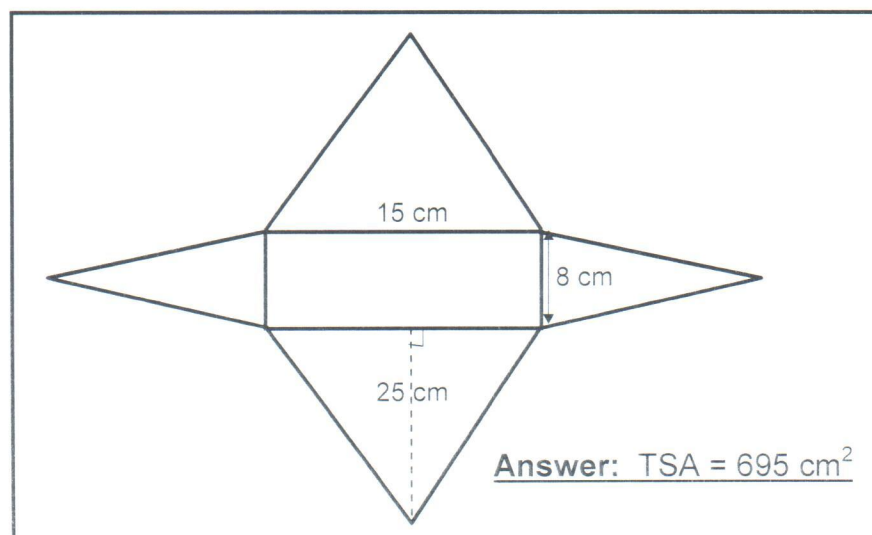
Answer: TSA = 663 cm^2

c)



Answer: TSA = 153 cm^2

d)



Answer: TSA = 695 cm^2

Practice Exercise 16

1. 477.52 cm^2
 2. 1036.73 cm^2
 3. 502.65 cm^2
 4. 867.08 cm^2
-

Practice Exercise 17

1. 4.5 m^2
 2. 6.8 m^2
 3. a) 10 m^2 b) 300 m^2 c) 270 m^2
d) 6.4 m e) 201.2 m^2
-

END OF TOPIC 3

TOPIC 4

VOLUME

Lesson 18:	Volume of Prism with Quadrilateral End Faces
Lesson 19:	Volume of Triangular Prisms
Lesson 20:	Volume of Cylinders
Lesson 21:	Volume of Pyramids and Cones
Lesson 22:	Volume and Problem Solving

TOPIC 4: VOLUME

Introduction



Have you ever wondered how one can tell the amount of water needed to fill an aquarium tank without even trying to put some water in it?

How about determining the amount of water needed to fill an Olympic size swimming pool?



You do not need to be an expert in order to measure the amount of liquid needed to fill in a tank or a pool.

As you go through this topic, you will learn that skills in solving for the volume of prisms, cylinders and cones will enable to measure the amount of water in an aquarium tank without getting wet.

Enjoy as you explore and learn the practical applications of the lessons in this Topic.

Lesson 18: Volume of Prism with Quadrilateral End Faces



You learnt about surface areas of solid figures in the previous topic.



In this lesson, you will:

- Identify the units of Volume and Capacity
- Calculate the volume of prism with one or more quadrilateral end faces.

Have you ever thought how much water an aquarium tank may hold?



Or ever wondered how much water a swimming pool may contain?

Or if you are in a business, what might be the dimension of your container to hold such an amount of liquid?

This lesson will give you a concrete idea and knowledge how to deal with such situations.

Does its sound interesting?

Yes it is interesting as it is useful in our everyday lives.

The **volume of a solid** is the amount of space it contains. **Volume** is measured in cubic units, such as **cubic feet (ft³)** and **cubic meters (m³)**.

Before we solve for the volume of solids, let us first recall the units of measurements used.

$$\begin{aligned}\text{Since } 1 \text{ cm}^3 &= 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} \\ 1 \text{ cm}^3 &= 1000 \text{ mm}^3\end{aligned}$$

How many cubic cm are there in 1 cubic meter?

$$\begin{aligned}\text{Since } 1 \text{ m}^3 &= 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \\ 1 \text{ m}^3 &= 1\,000\,000 \text{ cm}^3\end{aligned}$$

In dealing with volume, it is also important to take note of the units of capacity to be used in measuring the amount of space inside a container.

Units of Capacity

1 mL (capacity) has a volume of 1 cm^3 .

1 Litre (capacity) has a volume of 1000 cm^3

Since we have $1 \text{ mL} = 1 \text{ cm}^3$

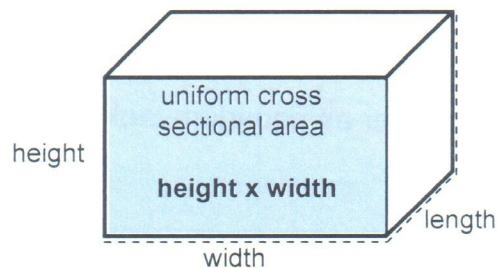
$1 \text{ mL} = 1 \text{ cm}^3$

$1 \text{ L} = 1000 \text{ cm}^3$

$1 \text{ L} = 1000 \text{ mL}$

Volume of Prism with Quadrilateral End Faces

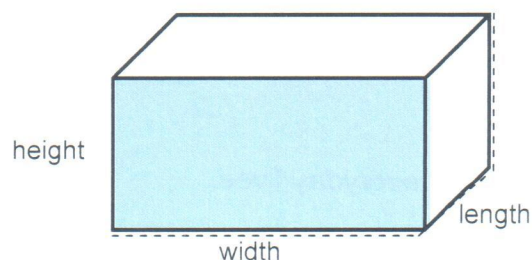
In the previous lessons we already discussed what a prism is. Let us again review its properties.



The **volume of a prism** with quadrilateral end faces is equal to the uniform cross sectional area times the length. This is the general rule we follow for every prism whose faces are quadrilaterals.

Let us now take a closer look at the different prisms and we derive a formula for each following the general rule in solving for the volume of prisms stated above.

Volume of Cuboids



The cross sectional area (CSA) of a rectangular prism or cuboid is equal to height x width.

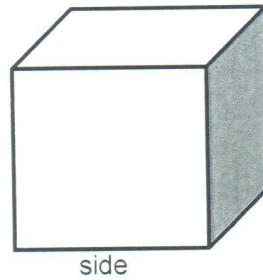
Therefore following the general rule applies:

$$V = \text{CSA} \times \text{length}$$

$$V = (\text{height} \times \text{width}) \times \text{length}$$

$$V = \text{height} \times \text{width} \times \text{length} \text{ or } lwh$$

Volume of a Cube



In a cube the length, width and height are congruent. Therefore we just represent them using s (side).

The cross sectional area (CSA) of a cube is equal to height $\times$ width but since the two are congruent we say side $\times$ side or s^2 .

Therefore, the general rule applies:

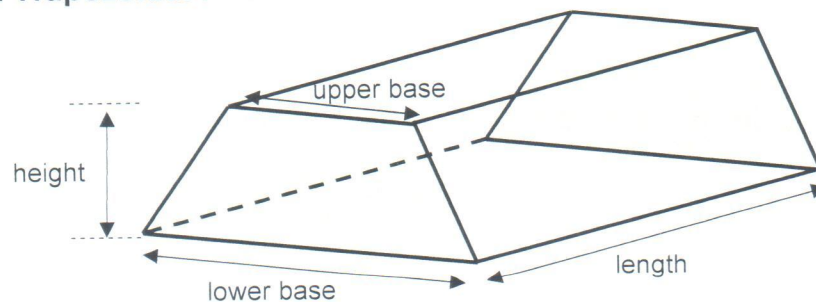
$$V = \text{CSA} \times \text{length}$$

But the length is also congruent with the width and height

$$V = \text{side} \times \text{side} \times \text{side}$$

$$V = (\text{side})^3 \text{ or } s^3$$

Volume of a Trapezoidal Prism



The cross sectional area (CSA) of a trapezoidal prism is equal to $\frac{1}{2} \times (\text{upper base} + \text{lower base}) \times \text{height}$ that is $\text{CSA} = \frac{1}{2}(\text{base}_{(\text{upper})} + \text{base}_{(\text{lower})}) (\text{height})$.

Therefore following the general rule applies:

$$V = \text{CSA} \times \text{length}$$

$$V = \frac{1}{2}(\text{base}_{(\text{upper})} + \text{base}_{(\text{lower})}) \times \text{height} \times \text{length}$$

$$V = \frac{1}{2}(\text{base}_{(\text{upper})} + \text{base}_{(\text{lower})}) (\text{height}) (\text{length})$$



Note that we applied the general rule in solving for the volume of the prisms with quadrilateral end faces. The differences in the formulas we derived are the cross sectional area.

Example 1

Find the volume and capacity of a cube whose side measures 22 cm.

Solution:

Since we already derived a simpler formula for the volume of a cube, we will now use

$$V = s^3.$$

Given: $s = 22 \text{ cm}$

Formula: $V = s^3$

Substitute the given in the formula: $V = (22 \text{ cm})^3$

Simplify: $V = (22 \text{ cm} \times 22 \text{ cm} \times 22 \text{ cm})$

$$V = 10648 \text{ cm}^3.$$

Solving for the cube's capacity:

Since we know that $1 \text{ mL} = 1 \text{ cm}^3$, then $10648 \text{ cm}^3 = 10648 \text{ mL}$

Therefore, the volume of a cube is 10648 cm^3 and its capacity is 10648 mL.

Example 2

What is the length of a cuboid given that its volume is 540 cm^3 and its width and height measures 5 cm and 9 cm respectively?

Solution:

Since we already derived a simpler formula for the volume of a cuboid, we will now use

$$V = \text{length} \times \text{width} \times \text{height}$$

Given: $V = 540 \text{ cm}^3$; $w = 5 \text{ cm}$; $h = 9 \text{ cm}$

Formula: $V = \text{length} \times \text{width} \times \text{height}$

Substitute the given in the formula: $540 \text{ cm}^3 = \text{length} \times 5 \text{ cm} \times 9 \text{ cm}$

Simplify: $540 \text{ cm}^3 = \text{length} \times 45 \text{ cm}^2$

Divide both sides of the equation by 45 cm^2

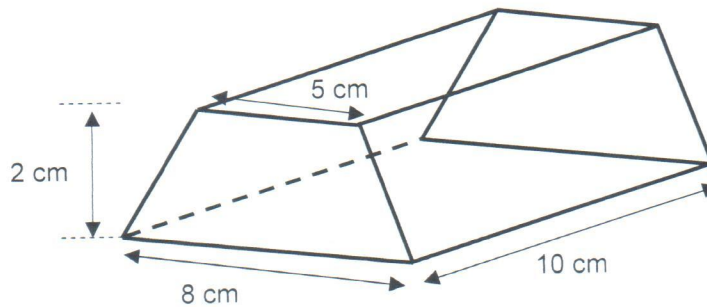
$$\frac{540 \text{ cm}^3}{45 \text{ cm}^2} = \frac{\text{length} \times 45 \text{ cm}^2}{45 \text{ cm}^2}$$

Simplify: $12 \text{ cm} = \text{length}$

Therefore, the length of the cuboid is 12 cm.

Example 3

What is the volume of the figure below?



Since we already derived a simpler formula for the volume of a trapezium prism, we will now use $V = \frac{1}{2}(\text{base}_{(\text{upper})} + \text{base}_{(\text{lower})})(\text{height})(\text{length})$.

Given: $h = 2 \text{ cm};$ $l = 10 \text{ cm}$
 $\text{base}_{(\text{upper})} = 5 \text{ cm};$ $\text{base}_{(\text{lower})} = 8 \text{ cm}$

Formula: $V = \frac{1}{2}(\text{base}_{(\text{upper})} + \text{base}_{(\text{lower})})(\text{height})(\text{length})$

Substitute the given in the formula: $V = \frac{1}{2}(5 \text{ cm} + 8 \text{ cm})(2 \text{ cm})(10 \text{ cm})$

Simplify: $V = \frac{1}{2}(13 \text{ cm})(2 \text{ cm})(10 \text{ cm})$

$$V = 130 \text{ cm}^3.$$

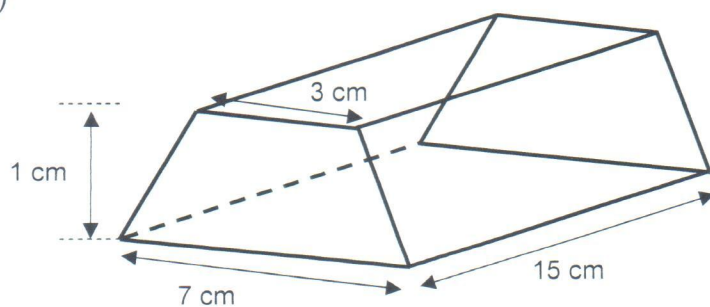
Therefore, the volume of the trapezium prism is 130 cm^3 .

NOW DO PRACTICE EXERCISE 18

**Practice Exercise 18**

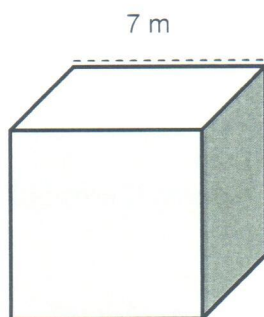
Find the volume and capacity in mL of the following prisms:

1)



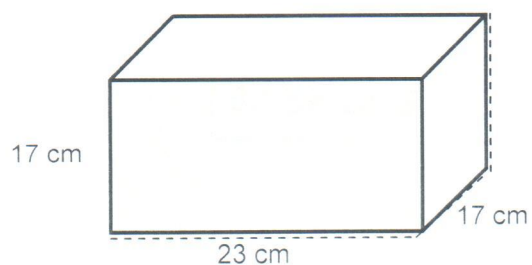
Answer: _____

2)



Answer: _____

3)



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 19: Volume of Triangular Prisms



You have learned about volume of prisms in Lesson 18.



In this lesson, you will:

- calculate the volume of triangular prism

Have you ever wondered how they determine the amount of chocolate to be put inside the container of this famous and mouth-watering chocolate?

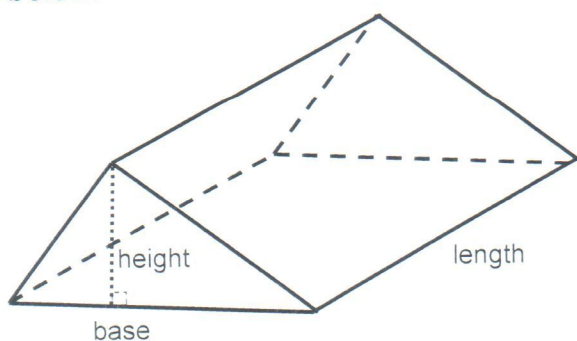


This lesson will help you learn how to solve for the volume of triangular prisms.

A triangular prism is a solid whose end is identical and whose faces are rectangles. The above figure is an example.

The general rule in solving for the volume of a cube also applies with this triangular prism.

Let us observe the figure below:



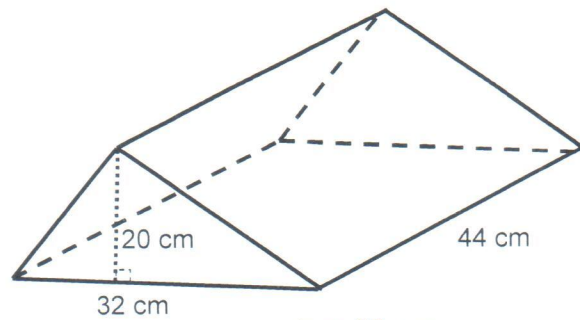
Following the general rule in solving for the volume of prisms we discussed in Lesson 18 we say that “volume of a prism is equal to the uniform cross sectional area times the length”.

The uniform cross sectional area (CSA) of a triangular prism is $A = \frac{1}{2} \times \text{base} \times \text{height}$

therefore, we say that : $V = \frac{1}{2} \times \text{base} \times \text{height} \times \text{length}$.

Example 1

Find the volume of the triangular prism below and solve for its capacity in litre (L).



Given:

$$h = 20 \text{ cm}; \quad b = 32 \text{ cm} \quad l = 44 \text{ cm}$$

Formula:

$$V = \frac{1}{2} \times \text{base} \times \text{height} \times \text{length}$$

Substitute the given in the formula:

$$V = \frac{1}{2} (32 \text{ cm}) (20 \text{ cm}) (44 \text{ cm})$$

Simplify:

$$V = 14\,080 \text{ cm}^3.$$

Solving for the triangular prism's capacity in litre (L) : we use $1 \text{ L} = 1000 \text{ cm}^3$

$$\frac{1 \text{ L}}{1000 \text{ cm}^3} = \frac{x}{14080 \text{ cm}^3}$$

$$\frac{1 \text{ L} (14080 \text{ cm}^3)}{1000 \text{ cm}^3} = x$$

Cross multiply:

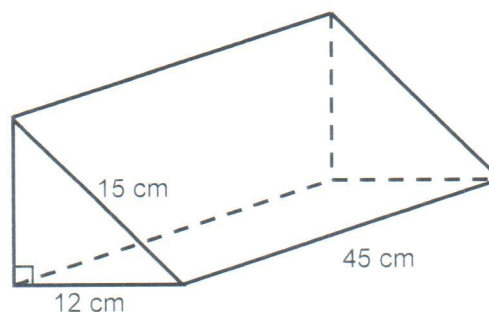
Simplify:

$$x = 14.08 \text{ L}$$

Therefore, the volume of the triangular prism is 14080 cm^3 and its capacity is 14.08 L.

Example 2

Using the figure below, solve for a) height b) CSA c) volume d) capacity in litre (L)



Solution:

a) height

Using the Pythagoras' Theorem, we can solve for the height

$$h = \sqrt{(15 \text{ cm})^2 - (12 \text{ cm})^2}$$

$$= \sqrt{225 \text{ cm}^2 - 144 \text{ cm}^2}$$

$$= \sqrt{81 \text{ cm}^2}$$

$$= 9 \text{ cm}$$

b) CSA

$$\begin{aligned}\text{CSA} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 12\text{cm} \times 9\text{cm} \\ &= 54\text{ cm}^2\end{aligned}$$

c) volume

$$\begin{aligned}V &= \text{CSA} \times \text{length} \\ &= 54\text{ cm}^2 \times 45\text{ cm} \\ &= \mathbf{2430\text{ cm}^3}\end{aligned}$$

d) capacity in litre (L) : we use $1\text{ L} = 1000\text{cm}^3$

$$\frac{1\text{ L}}{1000\text{cm}^3} = \frac{x}{2430\text{cm}^3}$$

Cross multiply: $\frac{1\text{ L}(\cancel{2430}\text{cm}^3)}{\cancel{1000}\text{cm}^3} = x$

Simplify: $x = \mathbf{2.43\text{ L}}$

Therefore, the triangular prism has: a) a height 9 cm b) a CSA of 54 cm^2 c) a volume of 2430 cm^3 and d) a capacity of 2.43 L.

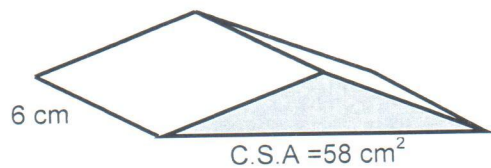
NOW DO PRACTICE EXERCISE 19



Practice Exercise 19

Find the volume and capacity in litre (L) of the following:

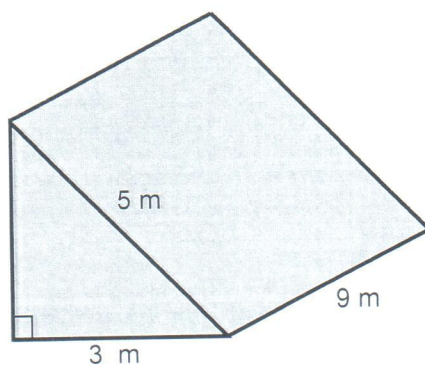
1)



Volume: _____

Capacity: _____

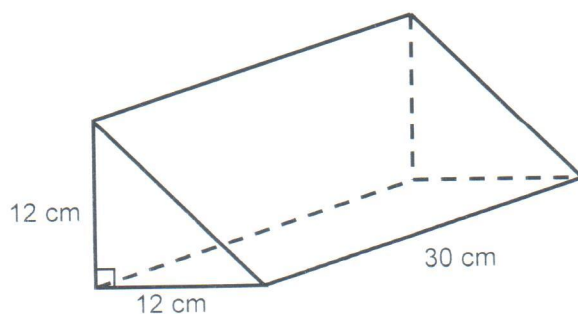
2)



Volume: _____

Capacity: _____

3)



Volume: _____

Capacity: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 20: Volume of Cylinders



You have learned something about volume of solids and area of cylinder in the previous lessons.

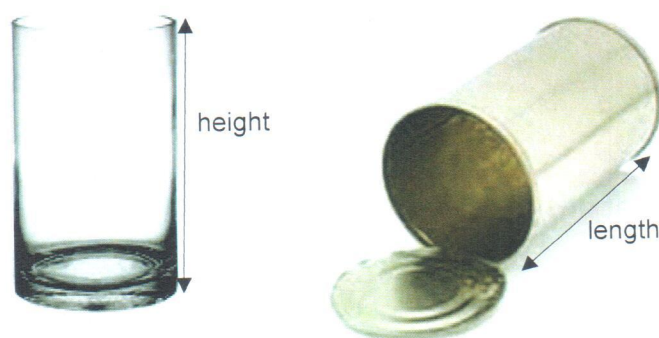


In this lesson you will:

- Calculate the volume of cylinders

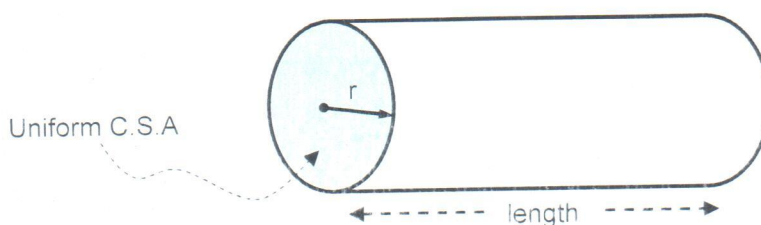
In Lesson 16, we have discussed about the surface area of cylinders and cylindrical objects. In this lesson, you will learn to measure the volume and the amount or capacity of liquid that may fit in a cylindrical container.

Observe the following containers, the amount of materials that make them has something to do with surface area but the amount of liquid that will fill in the glass and the tin can has something to do with volume.



In the above figures, we labelled the height of the glass as the vertical distance between the circular bases while the horizontal distance between the circular edges of the tin can was labelled as the length.

In this topic, for uniformity and consistency with the formula to be used, we will refer to **length as the distance between the two circular bases or edges**. It does not matter if the distance is vertical or horizontal.

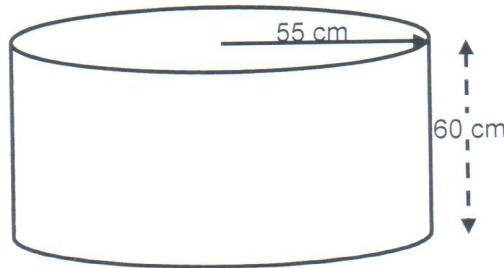


Getting the volume of the cylinder is not really complicated, just be reminded of the general rule in solving for the volume of solids which states that the volume is equal to the cross sectional area times the height.

Since the cross sectional area (CSA) of a cylinder is equal to the area of a circle with radius r , $A = \pi r^2$, we now write the formula for volume as **$V = \pi r^2 \times \text{length}$** .

Example 1

Find the volume of the cylindrical tank drawn below and solve for its capacity in litres (L).



Solution:

Given:

$$r = 55 \text{ cm} \quad l = 60 \text{ cm}$$

Formula:

$$V = \pi r^2 \times \text{length}$$

Substitute the given in the formula:

$$V = (3.14)(55\text{cm})^2 (60 \text{ cm})$$

Simplify:

$$V = (3.14)(3025\text{cm}^2)(60 \text{ cm})$$

$$\mathbf{V = 569\,910 \text{ cm}^3.}$$

Solving for the triangular prism's capacity in litre (L) : we use $1 \text{ L} = 1000\text{cm}^3$

$$\frac{1 \text{ L}}{1000\text{cm}^3} = \frac{x}{569910\text{cm}^3}$$

Cross multiply:

$$\frac{1\text{L}(\cancel{569910\text{cm}^3})}{\cancel{1000\text{cm}^3}} = x$$

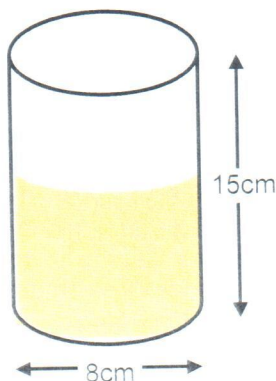
Simplify:

$$\mathbf{x = 569.91 \text{ L}}$$

Therefore, the volume of the cylindrical tank is $569\,910 \text{ cm}^3$ and its capacity is 569.91 L .

Example 2

Find the volume of the cylindrical glass below and determine the amount of juice in it if the glass is half-filled.



Solution:

Given: Since $d = 8$ cm, therefore $r = 4$ cm ; $l = 15$ cm

Formula: $V = \pi r^2 \times \text{length}$

Substitute the given in the formula: $V = (3.14)(4\text{cm})^2 (15 \text{ cm})$

Simplify: $V = (3.14)(16\text{cm}^2)(15\text{cm})$
 $V = 753.6 \text{ cm}^3.$

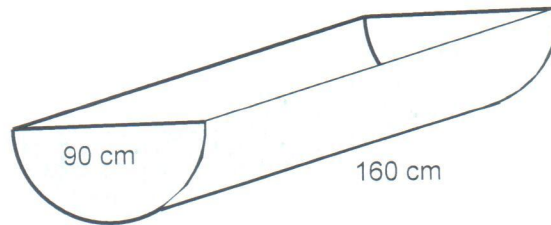
Solving for the cylindrical glass' capacity: we use $1 \text{ mL} = 1\text{cm}^3$

Since $1 \text{ mL} = 1 \text{ cm}^3$, the glass full capacity is 753.6 mL. Having it half-filled, we divide its full capacity by 2, $753.6 \text{ mL} \div 2 = 376.8 \text{ mL}$.

Therefore, the volume of the cylindrical glass is 753.6 cm^3 and the amount of juice it contains is 376.8 mL.

Example 3

A tub is in the shape of a half cylinder whose diameter is 90 cm and its length is 160 cm. What is its maximum capacity in litre (L)?



Solution:

Given: Since $d = 90$ cm, therefore $r = 45$ cm ; $l = 160$ cm

Formula: $V = \pi r^2 \times \text{length}$

But the tub is in the shape of a half cylinder, therefore, we multiply the formula by $\frac{1}{2}$.
 Instead we will use

$$V = \frac{1}{2} \pi r^2 \times \text{length}$$

Substitute the given in the formula: $V = \frac{1}{2} (3.14)(45\text{cm})^2 (160 \text{ cm})$

Simplify: $V = \frac{1}{2} (3.14)(2025\text{cm}^2)(160\text{cm})$

$$V = 508\,680 \text{ cm}^3.$$

Solving for the half cylindrical tub's capacity in litre (L) : we use $1 \text{ L} = 1000\text{cm}^3$

$$\frac{1\text{L}}{1000\text{cm}^3} = \frac{x}{508680\text{cm}^3}$$

Cross multiply:

$$\frac{1\cancel{\text{L}}(508680\cancel{\text{cm}^3})}{1000\cancel{\text{cm}^3}} = x$$

Simplify:

$$x = 508.68 \text{ L}$$

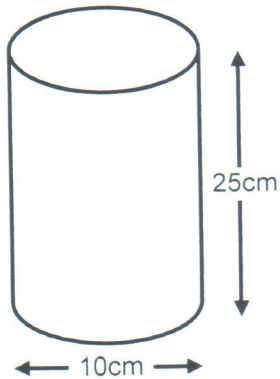
Therefore, the maximum capacity of the half-cylinder-shaped tub is 508.68 L.

NOW DO PRACTICE EXERCISE 20

**Practice Exercise 20**

Find the volume and capacity of the following in litres (L).

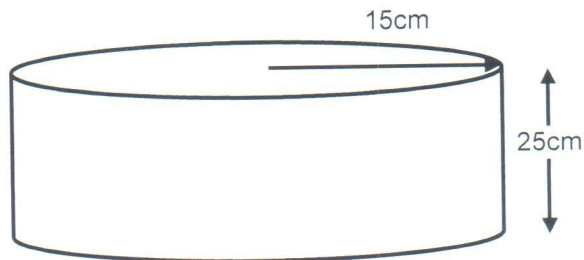
1)



Volume: _____

Capacity: _____

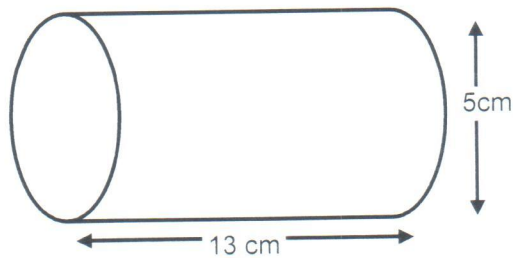
2)



Volume: _____

Capacity: _____

3)



Volume: _____

Capacity: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 21: Volume of Pyramids and Cones



You have learned something about the volume of cylinders in the previous lesson.

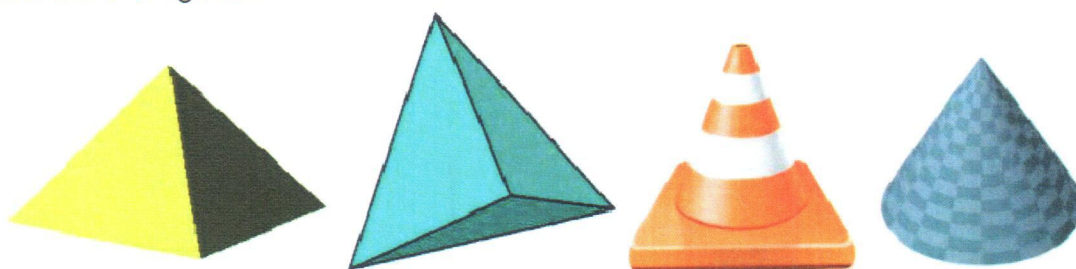


In this lesson, you will:

- calculate the volume of pyramids and cones.

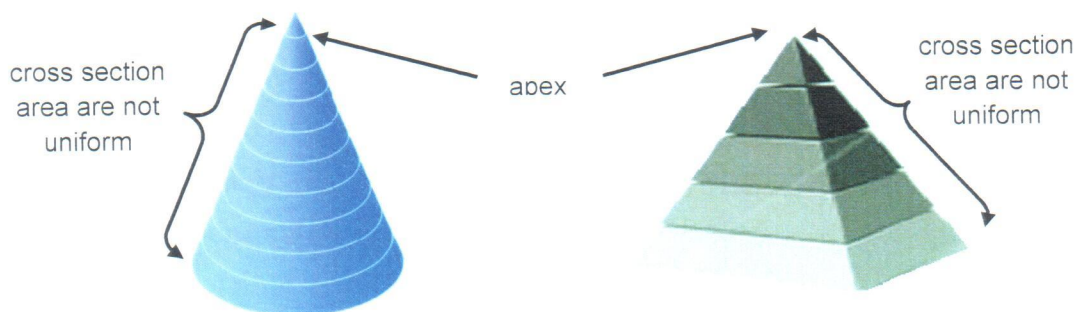
In the previous lessons in Topic 4 we were able to solve for the surface areas and volumes of solid figures following a general rule. In this topic we will deviate a little with the general rule in solving for the volume of most solids.

Consider the figures.



What did you notice among the figures drawn above as compared with the previous solids we have dealt with?

Very good observation if you say that, all the figures above have tips or pointed tips opposite their bases. These tips are called **apex**. Unlike other solid figures with two congruent faces opposite each other and makes a uniform cross sectional area, the cone and pyramid do not have CSA.



Notice that if the cone and pyramid are cut through, there will be no uniform cross section area.

This is the reason why the rule we followed in deriving for the formulas of volume of other solid figures cannot be applied for these two solid figures.

However, since the two have an apex and the computation for the volume is also founded on their bases a formula we use the same formula in solving for their volumes.

$$\text{Volume of a Pyramid or Cone} = \frac{1}{3} \times \text{area of the base} \times \text{height}$$

Volume of a Pyramid

Since a pyramid has different bases, the formula for each kind varies. Let us sum up using the following:

Square-based Pyramid

$$V = \frac{1}{3} \times \text{side}^2 \times \text{height}$$

Rectangular-based Pyramid

$$V = \frac{1}{3} \times \text{length} \times \text{width} \times \text{height}$$

Triangle-based Pyramid or tetrahedron

$$V = \frac{1}{3} \times \left(\frac{1}{2} \times \text{base} \times \text{height of the base} \right) \times \text{height of the pyramid}$$

$$V = \frac{1}{6} \times \text{base}_{\text{triangle}} \times \text{height}_{\text{base}} \times \text{height}_{\text{pyramid}}$$

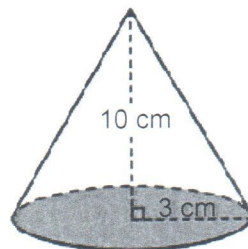
Volume of a Cone

Unlike the pyramid, the base of the cone is a circle and it only varies on its measure.

$$V = \frac{1}{3} \times \pi r^2 \times \text{height}$$

Example 1

Find the volume of the figure below:



Solution

Given: $h = 10 \text{ cm};$ $r = 3 \text{ cm}$

Formula: $V = \frac{1}{3} \times \pi r^2 \times \text{height}$

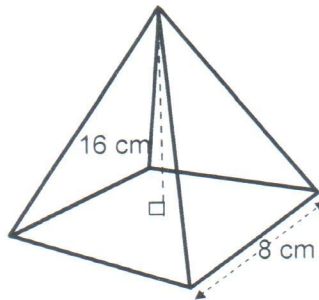
Substitute the given in the formula: $V = \frac{1}{3} (3.14) (3\text{cm})^2 (10\text{cm})$

Simplify: $V = 94.2\text{cm}^3.$

Therefore, the volume of the cone is 94.2 cm^3 .

Example 2

Find the volume and the capacity in litre (L) of the pyramid drawn below:



Solution

Given: $h = 16 \text{ cm}$; $s = 8 \text{ cm}$

Formula: $V = \frac{1}{3} \times \text{side}^2 \times \text{height}$

Substitute the given in the formula: $V = \frac{1}{3} (8\text{cm})^2 (16\text{cm})$

Simplify: $V = 341.33 \text{ cm}^3$.

Solving for the pyramid's capacity in litre (L): we use $1 \text{ L} = 1000\text{cm}^3$

$$\frac{1\text{L}}{1000\text{cm}^3} = \frac{x}{341.33\text{cm}^3}$$

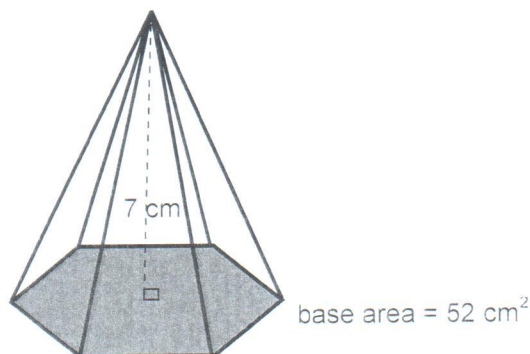
Cross multiply: $\frac{1\text{L}(341.33\text{cm}^3)}{1000\text{cm}^3} = x$

Simplify: $x = 0.34 \text{ L}$

Therefore, the volume of the square-based pyramid is 341.33 cm^3 and its capacity is 0.34 L .

Example 3

Find the volume of the pyramid drawn below.



Solution:

The given pyramid has a base of hexagon. Since the base area of the hexagon is given, we use the general formula $V = \text{area of the base} \times \text{height}$.

Given:

$$h = 7 \text{ cm}; \quad A_{\text{base}} = 52 \text{ cm}^2$$

Formula:

$$V = \text{area of the base} \times \text{height}$$

Substitute the given in the formula:

$$V = (52 \text{ cm}^2) (7 \text{ cm})$$

Simplify:

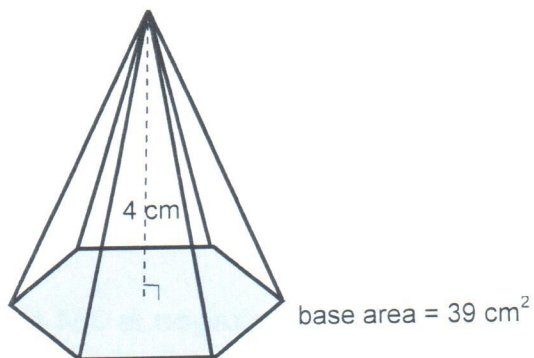
$$V = 364 \text{ cm}^3.$$

Therefore, the volume of the pyramid whose base is a hexagon is 364 cm^3 .

NOW DO PRACTICE EXERCISE 21

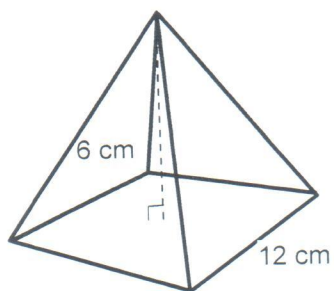
**Practice Exercise 21**

1.



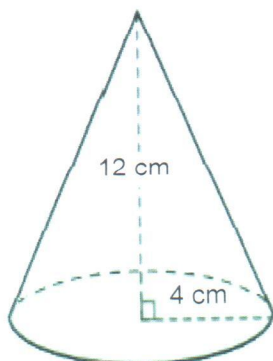
Answer: _____

2.



Answer: _____

3.



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 22: Volume and Problem Solving



You have learned something about the volumes of different solids in the previous lessons..



In this lesson, you will:

- solve general problems on volume of solids

Congratulations! You have reached the last lesson in this Unit book. At this point it is certain that you already have developed the confidence and the skill in solving for the volume of the different solid figures.



Remember that you need not to memorize all formulas, learn the general rules by heart and memorize the fundamentals these are the formulas for areas. Then you are ready to derive the formula you need in a given situation.

In this lesson, you will apply what you have learned in this topic by solving real-life problems.

Example 1

Below is a four-cylinder engine. Each cylinder has a radius of 6 cm and a height of 8 cm.

- Find the volume of one cylinder?
- What is the capacity of 1 cylinder in litre(L)?
- Calculate the total capacity of the engine in litre(L)?



Solution:

- Volume of one cylinder

Given: $r = 6 \text{ cm}$ $l = 8 \text{ cm}$

Formula:

$$V = \pi r^2 \times \text{length}$$

Substitute the given in the formula:

$$V = (3.14)(6\text{cm})^2(8 \text{ cm})$$

Simplify:

$$V = (3.14)(36\text{cm}^2)(8\text{cm})$$

$$V = 904.32 \text{ cm}^3.$$

b) Capacity of 1 cylinder

Solving for the cylinder's capacity in litre (L) : we use $1 \text{ L} = 1000 \text{ cm}^3$

$$\frac{1 \text{ L}}{1000 \text{ cm}^3} = \frac{x}{904.32 \text{ cm}^3}$$

Cross multiply:

$$\frac{1 \text{ L}(904.32 \text{ cm}^3)}{1000 \text{ cm}^3} = x$$

Simplify:

$$x = 0.90 \text{ L}$$

c) Capacity of the engine

Since the engine has 4 cylinders, we multiply the capacity of 1 engine by 4

$$\text{Total capacity of the engine} = 0.90 \text{ L} \times 4 = 3.6 \text{ L}$$

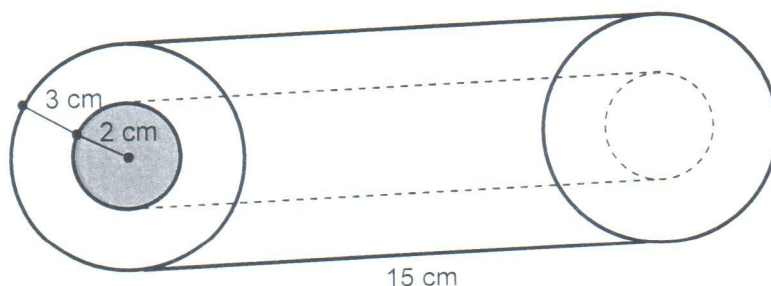
Therefore, the engine has a total capacity of 3.6 L where each cylinder has a volume of 904.32 cm^3 with a capacity of 0.90 L.

Example 2

A steel manufacturer produced a metal cylinder with a cylindrical hole drilled through it. The dimensions are shown in the figure below.

a) Solve for the volume of the cylindrical metal object

b) Find the weight of the cylindrical metal object if 1 cm^3 weighs 8 grams.



Solution:

a) Since the cylindrical metal object is hallowed inside, we have to get the volume of the large cylinder and subtract the volume of the hallow cylinder.

$$V = \text{volume of the large cylinder} - \text{volume of the hallow cylinder}$$

Volume of the large cylinder

Given:

$$r = 5 \text{ cm} \quad l = 15 \text{ cm}$$

Formula:

$$V = \pi r^2 \times \text{length}$$

Substitute the given in the formula:

$$V = (3.14)(5 \text{ cm})^2(15 \text{ cm})$$

Simplify:

$$V = (3.14)(25 \text{ cm}^2)(15 \text{ cm})$$

$$V = 1177.5 \text{ cm}^3.$$

Volume of the hallow cylinder

Given: $r = 2 \text{ cm}$ $l = 15 \text{ cm}$

Formula: $V = \pi r^2 \times \text{length}$

Substitute the given in the formula: $V = (3.14)(2\text{cm})^2(15 \text{ cm})$

Simplify: $V = (3.14)(4\text{cm}^2)(15\text{cm})$

$$\mathbf{V = 188.4 \text{ cm}^3.}$$

Volume of the metal cylinder (V)

$V = \text{volume of the large cylinder} - \text{volume of the hallow cylinder}$

$$V = 1177.5 \text{ cm}^3 - 188.4 \text{ cm}^3$$

$$\mathbf{V = 989.1 \text{ cm}^3}$$

b) Weight of the metal cylinder

Since $1 \text{ cm}^3 = 8 \text{ grams}$

We use the proportion: $\frac{1 \text{ cm}^3}{8 \text{ grams}} = \frac{989.1 \text{ cm}^3}{\text{weight}}$

Cross multiply: $\text{weight} = \frac{(989.1 \text{ cm}^3)(8 \text{ grams})}{1 \text{ cm}^3}$

Simplify: $\text{weight} = 7912.8 \text{ grams}$

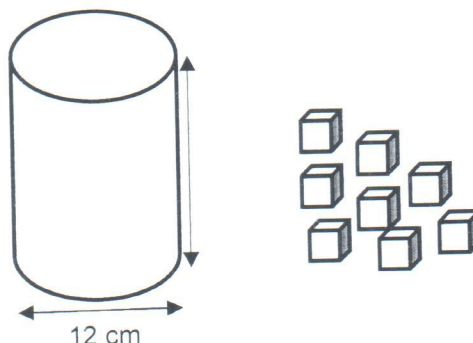
Therefore, the volume of the metal cylinder is 989.1 cm^3 and it weighs 7912.8 grams.

NOW DO PRACTICE EXERCISE 22

**Practice Exercise 22**

Solve the following problems.

1. A cylindrical water bottle is to be filled with water and 8 cubed ice. Each cube ice has a side length of 2 cm.



- a) What is the volume of the water bottle?

Answer: _____

- b) What is the maximum capacity of the water bottle in mL?

Answer: _____

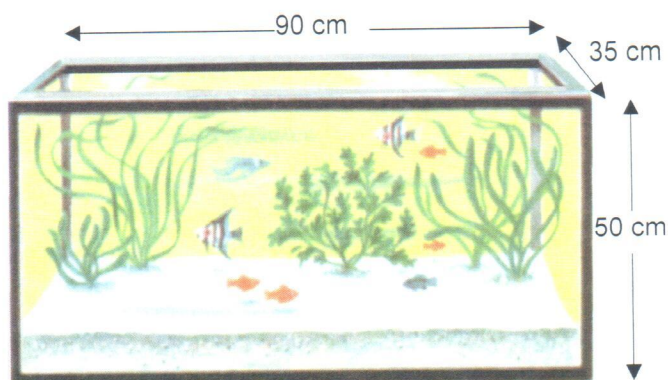
- c) What is the volume of each cubed ice?

Answer: _____

- d) If all the cubed ice are to be placed inside the water bottle, how much more water can the bottle hold? (express answer in mL)

Answer: _____

2. Buddy would like to set up a fish tank in his room. The salesman in the pet shop reminded him that he has to put a maximum of $\frac{3}{4}$ water to accommodate the other ornaments he will put in his aquarium.



- a) What is the volume of the fish tank?

Answer: _____

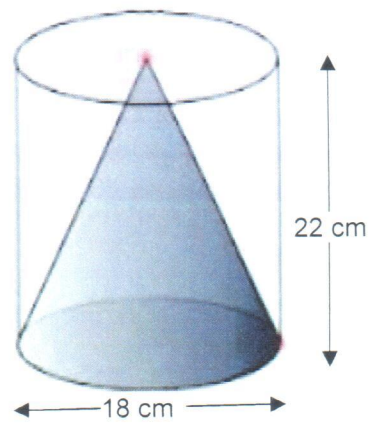
- b) What is the maximum capacity of the tank in (L) if it will be filled with water?

Answer: _____

- c) What is the maximum capacity in litre (L) of the tank if it will only be filled with $\frac{3}{4}$ water?

Answer: _____

3. Use the figure below to answer the questions that follow.



- a) What is the volume of the cylinder?

Answer: _____

- b) What is the volume of the cone?

Answer: _____

- c) How much more capacity in mL can the cylinder hold?

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

TOPIC 4: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- The volume of a solid is the amount of space it contains. Volume is measured in cubic units, such as cubic feet (ft³) and cubic meters (m³).
- The volume of a prism with quadrilateral end faces is equal to the uniform cross sectional area times the length. This is the general rule we follow for every prism whose faces are quadrilaterals.
- Volume of a Pyramid or Cone = $\frac{1}{3}$ x area of the base x height
- A triangular prism is a solid whose end is identical and whose faces are rectangles. The above figure is an example.

REVISE LESSONS 18 - 22. THEN DO TOPIC TEST 4 IN ASSIGNMENT BOOK 6.

ANSWERS TO PRACTICE EXERCISES 18 - 22

Practice Exercise 18

- | | | | |
|----|----------------------|-------------------------|-------------------------|
| 1. | a) 75 cm^2 | b) 343 m^2 | c) 6647 cm^2 |
| 2. | a) 10 m^2 | b) 300 m^2 | c) 270 m^2 |
| | d) 6.36 m | e) 236.48 m^2 | f) 276.48 m^2 |
-

Practice Exercise 19

1. Volume = 348 cm^3 , Capacity = 0.348 L
 2. Volume = 54 m^3 or $540\,000 \text{ cm}^3$, Capacity = 540 L
 3. Volume = 2160 cm^3 , Capacity = 2.16 L
-

Practice Exercise 20

1. Volume = 1962.5 cm^3 , Capacity = 1.96625 L
 2. Volume = $17\,662.5 \text{ cm}^3$, Capacity = 17.6625 L
 3. Volume = 255.125 cm^3 , Capacity = 0.255125 L
-

Practice Exercise 21

1. Volume = 156 cm^3
 2. Volume = 288 cm^3
 3. Volume = 200.96 cm^3
-

Practice Exercise 22

- | | | | | |
|----|---------------------------|---------------------------|------------------------------------|-----------------------|
| 1. | a) $22\,608 \text{ cm}^3$ | b) $22\,608 \text{ mL}$ | c) $8 \text{ cm}^3 = 8 \text{ mL}$ | d) 22616 mL |
| 2. | a) 157.5 L | b) 118.125 L | c) 276.625 L | d) 22616 mL |
| 3. | a) 5595.48 cm^3 | b) 1865.16 cm^3 | c) 3730.32 m | |
-

END OF UNIT

REFERENCES

Bearing pdf file

http://maths.qahs.org.uk/files/2014/05/Trigonometry_bearings.pdf

Trigonometric worksheet – Bearing

http://ww2.valdosta.edu/~dboyd/Math1112/bearings_worksheet.pdf

Year 7 Interactive Maths - Second Edition

http://www.mathsteacher.com.au/year7/ch08_angles/07_bear/bearing.html

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SUBJECT AND GRADE TO STUDY

Grade Levels	Subjects
Grades 7 and 8	1. English
	2. Mathematics
	3. Science
	4. Social Science
	5. Making a Living
	6. Personal Development
	7. English
Grades 9 and 10	1. English
	2. Formal Mathematics
	3. Practical Mathematics
	4. Science
	5. Social Science
	6. Commerce
	7. Design and Technology- Computing
	8. Personal Development
Grades 11 and 12	1. English
	• Applied English
	• Language and Literature
	2. Mathematics
	• Mathematics A
	• Mathematics B
	3. Science
	4. Social Science
	• History
	• Geography
	• Economics
	5. Business Studies
	6. Personal Development
	7. ICT

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) courses.
- For Grades 9 and 10, you must study English, Mathematics, Science, Personal Development, Social Science and Commerce. Design and Technology-Computing is optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) courses to be certified. Your Provincial Coordinator or Supervisor will give you more information regarding each subject.

Certificate in Matriculation

CORE COURSES

Basic English
English 1
English 2
Basic Maths
Maths 1

Maths 2

History of Science & Technology

OPTIONAL COURSES

Science Streams: Biology

Chemistry, Physics and Social Science Streams:

Geography, Introduction to Economics and Asia and the Modern World

REMEMBER:

You must successfully complete 8 courses; 5 compulsory and 3 optional